2.8 Code specifications for beam columns

Code provisions on stability of beam-columns can be regarded under the light of the general theoretical bases outlined within the previous sections whose contents are a fundamental background for whatever design-oriented discussion on stability of members and structures.

In the present paragraph the basic provisions of two codes of standards of interest for designers mainly working in Italy and Europe will be described and commented with reference to the key aspects related to stability checks of steel members. Indeed, the basic provisions of both the Italian Code (D.M. 96 [12]) and the European one (Eurocode 3, [13] and [14]) will be analyzed.

Global stability will be basically discussed even if some insights above local buckling possibly affecting steel members will be even addressed with particular reference to the main rules provided by Eurocode 3.

2.8.1 Stability check of beam-columns according to Eurocode 3

A more general approach to stability of beam-columns is pursuit in Eurocode 3 in which a clear difference is firstly stated among steel members depending on the geometric and mechanical properties of their cross section. Consequently, steel profiles are divided into four different classes with respect to their ultimate behaviour in terms of both (flexural) strength and ductility. The first of the following subsections will explain with enough details this issue, while the other two will deal with the stability check of members under axial load with or without eccentricity.

Finally, it is worth noticing that safety checks in EC3 are formulated within the general framework of the Limit States Methods. Consequently, stability check deals with the Ultimate Limit State of the member and the action can be combined according to a rule like the one reported by equation (2.72). Similar formula can be even assumed for defining the design strength of material, but a more complicated way of defining the value of partial safety factor is considered in EC3, in which a different value has to be taken into account depending on the kind of failure mechanism of concern. Precisely, a value $\gamma_{M1}=1.05$ can be assumed for Stability while all the other possible values of the safety factors are reported in Table 2.1 for the sake of completeness.

Table 2.1: Values of the safety factors according to EC3.

LIMIT STATE	SUBJECT	SAFETY FACTORS
MATERIALE	Elastic limit state of the section	γ_M=1.00
	Plastic collapse of the structure	γ_M=1.12
	Transverse section in Class 1, 2, 3	γ_{M0}=1.05
	Transverse section in Class 4	γ_{M1}=1.05
	Stability of members	γ_{M1}=1.05
	Strength of net section	γ_{M2}=1.20
JOINTS	Bolts	γ_{Mb}=1.35
	Rivets	γ_{Mr}=1.35
	Pivots	γ_{Mp}=1.35
	Angle welds	γ_{Mw}=1.35
	Weldings in first class	γ_{Mw}=1.05
	Weldings in secondo class	γ_{Mw}=1.20
FRICTION JOINTS	Ultimate Limit State	$\gamma_{Ms,ult}$=1.25
	Serviceability Limit State	$\gamma_{Ms,ser}$=1.25
	Ultimate Limit State (lerger holes)	$\gamma_{Ms,ult}$=1.50
FATIGUE	Fatice strength	γ_{Mf}=1.00
FRAGILITY	Non welded	γ_{C1}=1.00
	Welded	γ_{C2}=1.50

2.8.1.1 Classification of steel cross sections

Behaviour of structural steel is ductile in behaviour since huge values of ultimate strain can be developed, namely in tension, after yielding. Nevertheless, slenderness of both section and member as a whole can undermine the natural ductility of steel as compression arises. Since, the above discussion mainly focused upon the global buckling issue, the present section will address the aspects related to local buckling of steel members which hugely affects their flexural behaviour. In particular, four different classes of profiles can be defined looking after their behaviour in terms of sectional strength and ductility, qualitatively depicted in Figure 2.14 in terms of moment-curvature relationships.

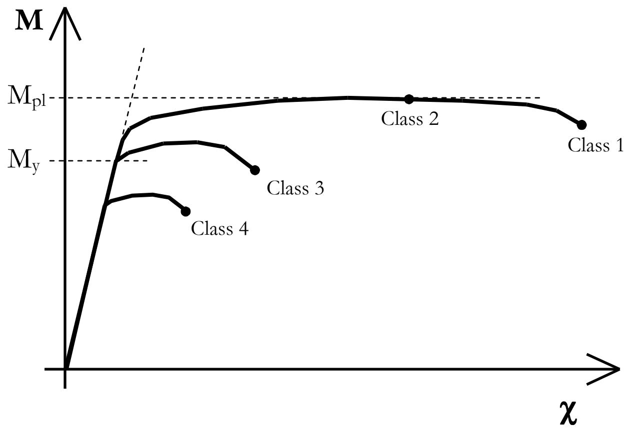


Figure 2.14: Possible moment-curvature behaviour for steel profiles.

In particular, the four classes corresponding to the various responses represented in the mentioned figure will be defined as follows:

- Class 1 cross-sections are those can form a plastic hinge with the rotation capacity required for plastic analysis;
- Class 2 cross-sections are those which can develop their plastic moment resistance, but have limited rotation capacity;
- Class 3 cross-section are those in which the calculated stress in the extreme compression fibre of the steel member can reach its yield stress, but local buckling is liable to prevent development of the plastic moment resistance;
- Class 4 cross-sections are those in which local buckling will occur before the attainment of yield stress in one or more parts of the cross-sections.

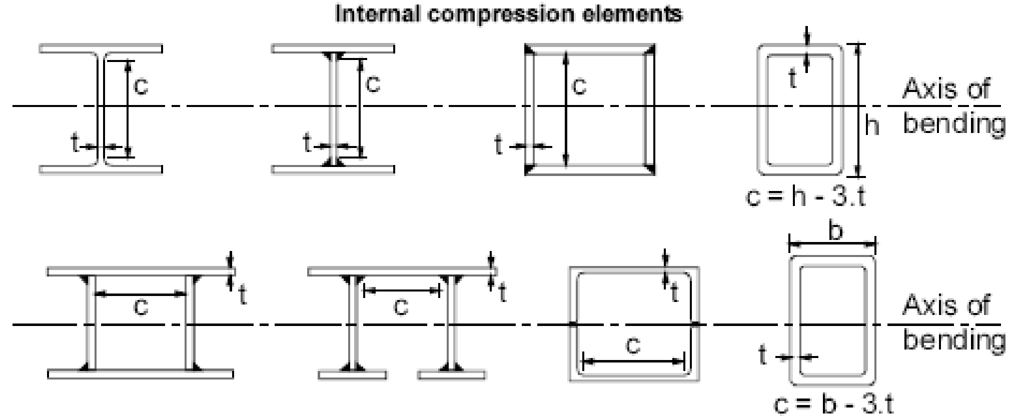
Beam-column sections can be classified into one of the above classes depending upon the three aspects listed below:

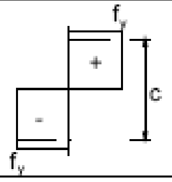
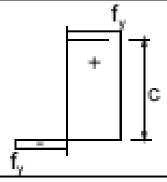
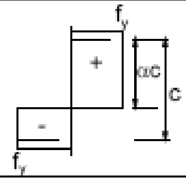
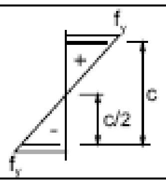
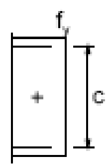
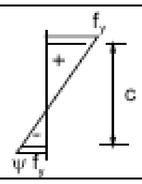
- effective length-to-thickness d/t of web and free length-to-thickness b/t of flanges;
- state of stress (pure bending, pure compression or bending and compression);
- grade of steel, being stronger steels more sensitive to local buckling phenomenon than weaker ones.

Table 2.2 summarizes the above concepts with reference to the most common hot-rolled I-shaped sections; similar tables can be found in EC3 for other joist sections.

Table 2.2: Classification of sections for local buckling according to EC3: webs.

Internal compression elements

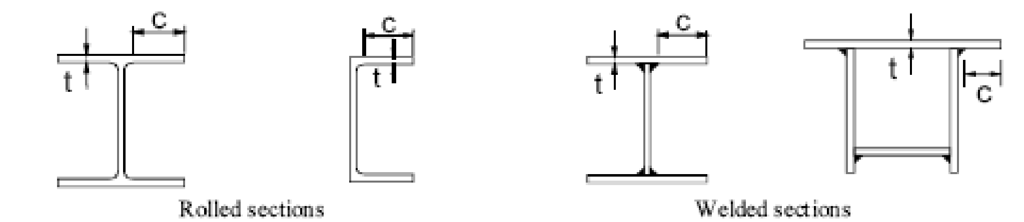


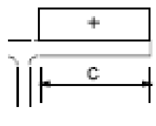
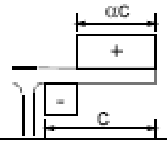
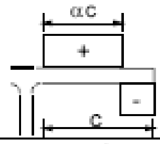
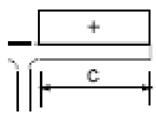
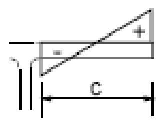
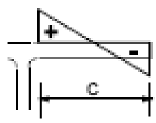
Class	Element subject to bending	Element subject to compression	Element subject to bending and compression
Stress distribution in elements (compression positive)			
1	$c/t \leq 72\varepsilon$	$c/t \leq 33\varepsilon$	when $\alpha > 0,5$: $c/t \leq \dfrac{396\varepsilon}{13\alpha - 1}$ when $\alpha \leq 0,5$: $c/t \leq \dfrac{36\varepsilon}{\alpha}$
2	$c/t \leq 83\varepsilon$	$c/t \leq 38\varepsilon$	when $\alpha > 0,5$: $c/t \leq \dfrac{456\varepsilon}{13\alpha - 1}$ when $\alpha \leq 0,5$: $c/t \leq \dfrac{41,5\varepsilon}{\alpha}$
Stress distribution in elements (compression positive)			
3	$c/t \leq 124\varepsilon$	$c/t \leq 42\varepsilon$	when $\psi > -1$: $c/t \leq \dfrac{42\varepsilon}{0,67 + 0,33\psi}$ when $\psi \leq -1^{*)}$: $c/t \leq 62\varepsilon(1-\psi)\sqrt{(-\psi)}$

$\varepsilon = \sqrt{235/f_y}$	f_y	235	275	355	420	460
	ε	1,00	0,92	0,81	0,75	0,71

Table 2.3: Classification of sections for local buckling according to EC3: flanges.

Outstand flanges



Class	Element subject to compression	Element subject to bending and compression: Tip in compression	Element subject to bending and compression: Tip in tension
Stress distribution in elements (compression positive)			
1	$c/t \leq 9\varepsilon$	$c/t \leq \frac{9\varepsilon}{\alpha}$	$c/t \leq \frac{9\varepsilon}{\alpha\sqrt{\alpha}}$
2	$c/t \leq 10\varepsilon$	$c/t \leq \frac{10\varepsilon}{\alpha}$	$c/t \leq \frac{10\varepsilon}{\alpha\sqrt{\alpha}}$
Stress distribution in elements (compression positive)			
3	$c/t \leq 14\varepsilon$	$c/t \leq 21\varepsilon\sqrt{k_\sigma}$ For k_σ see Part 1-5	

$\varepsilon = \sqrt{235/f_y}$	f_y	235	275	355	420	460
	ε	1,00	0,92	0,81	0,75	0,71

The web and the flanges of the section can be classified according to their dimensions, the steel grade and the state of stress according to the rules briefly reported in Table 2.2 and Table 2.3. The section as a whole has to be classified in class of the most slender of its members. If such a section falls in Class 4, local stability occurs before of yielding moment and, consequently, the flexural strength of the member cannot be completely developed. For this reason an effective section have to be determined by reducing the compressed area of web and flange in order to obtain a reduced virtual section to be considered in both strength and stability check. The way in which such section can be determined are not completely explained within this notes for the sake of brevity; nevertheless, the reader could directly refer to Eurocode 3 (section 6.2.2.5) for this topic.

2.8.1.2 Stability check under axial load

Stability check under axial load can be carried out through the following inequality:

$$N_{Sd} \leq N_{Rd} = \chi_{\min} \beta_A A \frac{f_{ay}}{\gamma_{M1}} , \tag{2.59}$$

being $\chi_{\min} = \min\left(\chi(\bar{\lambda}_x);\chi(\bar{\lambda}_y)\right)$ a reduction factor related to the relative slenderness $\bar{\lambda}$ defined as follows:

$$\bar{\lambda} = \sqrt{\frac{\beta_A A f_y}{N_{cr}}} . \tag{2.60}$$

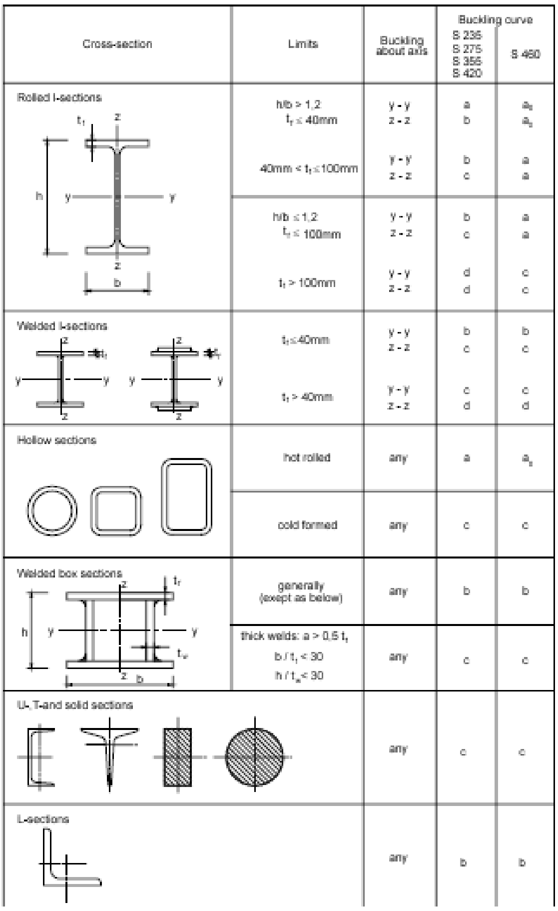
The factor β_A is defined as a ratio between the effective area and the gross section area; for section belonging to the first three classes defined above $\beta_A = 1$, while values smaller than the unity characterize profiles in class 4. Further details about the definition of the effective area for Class 4 profiles will be given in the following. If $\beta_A = 1$, the following relationship can be stated between the relative slenderness $\bar{\lambda}$, the absolute one λ and the critical one λ_p:

$$\bar{\lambda} = \sqrt{\frac{Af_y}{\pi^2 EI} \cdot L_0^{\ 2}} = \sqrt{\frac{f_y}{\pi^2 E} \cdot \frac{L_0^{\ 2}}{\rho^2}} = \frac{\lambda}{\lambda_p} \ . \tag{2.61}$$

Under a conceptual standpoint the parameter χ is basically the inverse of the ω factor reducing plastic strength of the section for looking after the global slenderness λ on the column. The relationship $\omega(\bar{\lambda})$ depends once more by the kind of imperfections and, consequently, by the type of profile. Four curves denoted as *a*, *b*, *c* and *d* can be utilized for that relationship:

- Curve a represents quasi perfect shapes: hot-rolled I-sections (h/b>1,2) with thin flanges (t_f≤40mm) if buckling is perpendicular to the major axis; it also represents hot-rolled hollow sections;
- Curve b represents shapes with medium imperfections: it defines the behaviour of most welded box-sections; of hot-rolled I-sections buckling about the minor axis; of welded I-sections with thin flanges (t_f > 40mm) and of the rolled I-sections with medium flanges (40<t_f≤100mm) if buckling is about the major axis; it also concerns cold-formed hollow sections where the average strength of the member after forming is used;
- Curve c represents shapes with a lot of imperfections: U, L, and T shaped sections are in this category as are thick welded box-sections; cold-formed hollow sections designed to the yield strength of the original sheet; hot-rolled H-sections (h/b ≤ 1,2 and t_f ≤ 100mm) buckling about the minor axis; and some welded I-sections (t_f ≤ 40mm buckling about the minor axis and t_f > 40mm buckling about the major axis);
- Curve d represents shapes with maximum imperfections: it is to be used for hot-rolled I-sections with very thick flanges (t_f > 100 mm) and thick welded I-sections (t_f > 40 mm), if buckling occurs in the minor axis.

Figure 2.15 shows how to choose the right curve for each shape and bending direction according to the properties mentioned above.



Cross-section	Limits	Buckling about axis	Buckling curve S 235 S 275 S 355 S 420	Buckling curve S 460
Rolled I-sections	h/b > 1,2; t_f ≤ 40mm	y - y	a	a_0
		z - z	b	a_0
	40mm < t_f ≤ 100mm	y - y	b	a
		z - z	c	a
	h/b ≤ 1,2; t_f ≤ 100mm	y - y	b	a
		z - z	c	a
	t_f > 100mm	y - y	d	c
		z - z	d	c
Welded I-sections	t_f ≤ 40mm	y - y	b	b
		z - z	c	c
	t_f > 40mm	y - y	c	c
		z - z	d	d
Hollow sections	hot rolled	any	a	a_0
	cold formed	any	c	c
Welded box sections	generally (exept as below)	any	b	b
	thick welds: a > 0,5 t_f; b / t_f < 30; h / t_w < 30	any	c	c
U-, T-and solid sections		any	c	c
L-sections		any	b	b

Figure 2.15: Stability curves for the various kinds of profiles.

Once the right curve has been chosen the value of χ can be determined; different curve have to be used for determining the corresponding χ values with reference to the two principal axis of the section.

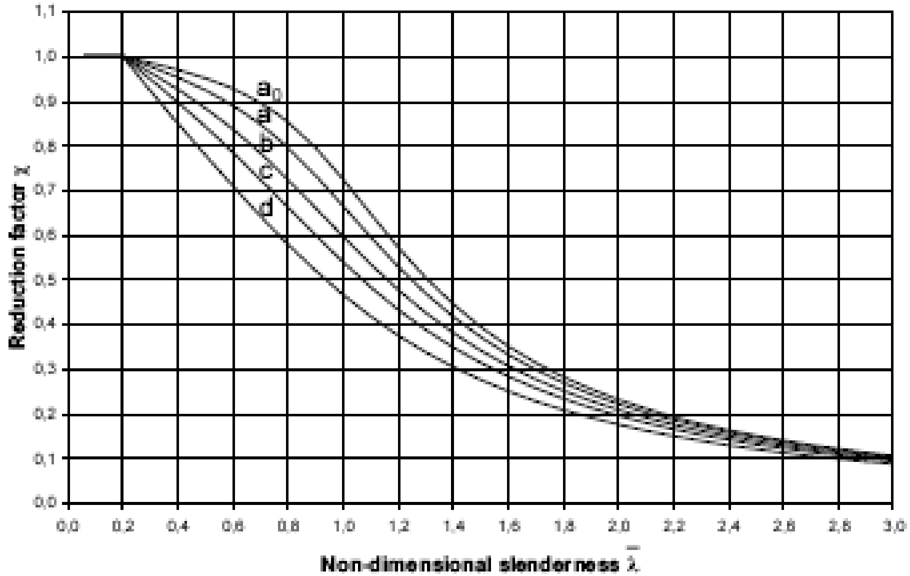


Figure 2.16: Stability curves according to EC3.

The curves in Figure 2.16 can be are defined by the following general relationship in terms of the non-dimensional slenderness defined by equation (2.60):

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}} \leq 1.0 \ , \qquad (2.62)$$

with

$$\Phi = 0.5 \cdot \left[1 + \alpha \cdot \left(\bar{\lambda} - 0.2 \right) + \bar{\lambda}^2 \right] \ . \qquad (2.63)$$

The parameter α is related to the level of imperfection affecting the structural member; since four curves have been introduced for describing the various kinds of imperfections, four values of α have to be considered, each one for the corresponding stability curve (**Errore. L'origine riferimento non è stata trovata.**).

Table 2.4: Imperfection factors for buckling curves.

Buckling curve	a_0	a	b	c	d
Imperfection factor α	0,13	0,21	0,34	0,49	0,76

2.8.1.3 Stability check under eccentric axial load

Since there slight differences exist between the method for stability check of members under eccentric loads according to the ENV version [7] and the final EN one [14], both procedures will be proposed in the following. Indeed, while the first one is not yet valid, in the authors' opinion, it is more suitable for grasping the mechanical meaning of the various terms whose final EN formalization seems only formally more complicated. Moreover, since in the symbols adopted within the Eurocodes, y and z are the two principal axes of inertia, this choice will be assumed in the following sections.

2.8.1.3.1 ENV 1-1-1993 [13] procedure.

Different formulae are provided depending on the class of the cross section. If it belongs to Classes 1 or 2 the following relationship, conceptually close to the one in equation (2.76) has to be considered:

$$\frac{N_{Sd}}{\frac{\chi_{\min} A f_{ay}}{\gamma_{M1}}} + \frac{k_y M_{y,Sd}}{\frac{W_{pl,y} f_{ay}}{\gamma_{M1}}} + \frac{k_z M_{z,Sd}}{\frac{W_{pl,z} f_{ay}}{\gamma_{M1}}} \le 1 \ , \tag{2.64}$$

where the coefficient $\chi_{\min} = \min\left(\chi(\bar{\lambda}_y);\chi(\bar{\lambda}_z)\right)$ can be evaluated according to the above remarks. Plastic moduli $W_{pl,y}$ and $W_{pl,z}$ are considered since plastic bending moment can be completely developed in class 1 and 2 profiles. Second-order effects and the shape of diagram are considered through the factors k_y and k_z, the first of which is defined as follows:

$$k_y = 1 - \frac{\mu_y N_{Sd}}{\chi_y A f_{ay}} \ , \tag{2.65}$$

and

$$\mu_y = \bar{\lambda}_y \cdot \left(2\beta_{My} - 4\right) + \frac{W_{pl,y} - W_{el,y}}{W_{el,y}} \ , \tag{2.66}$$

and, finally, the value of β_M accounts for the shape of bending moments and can be deduced by Figure 2.17.

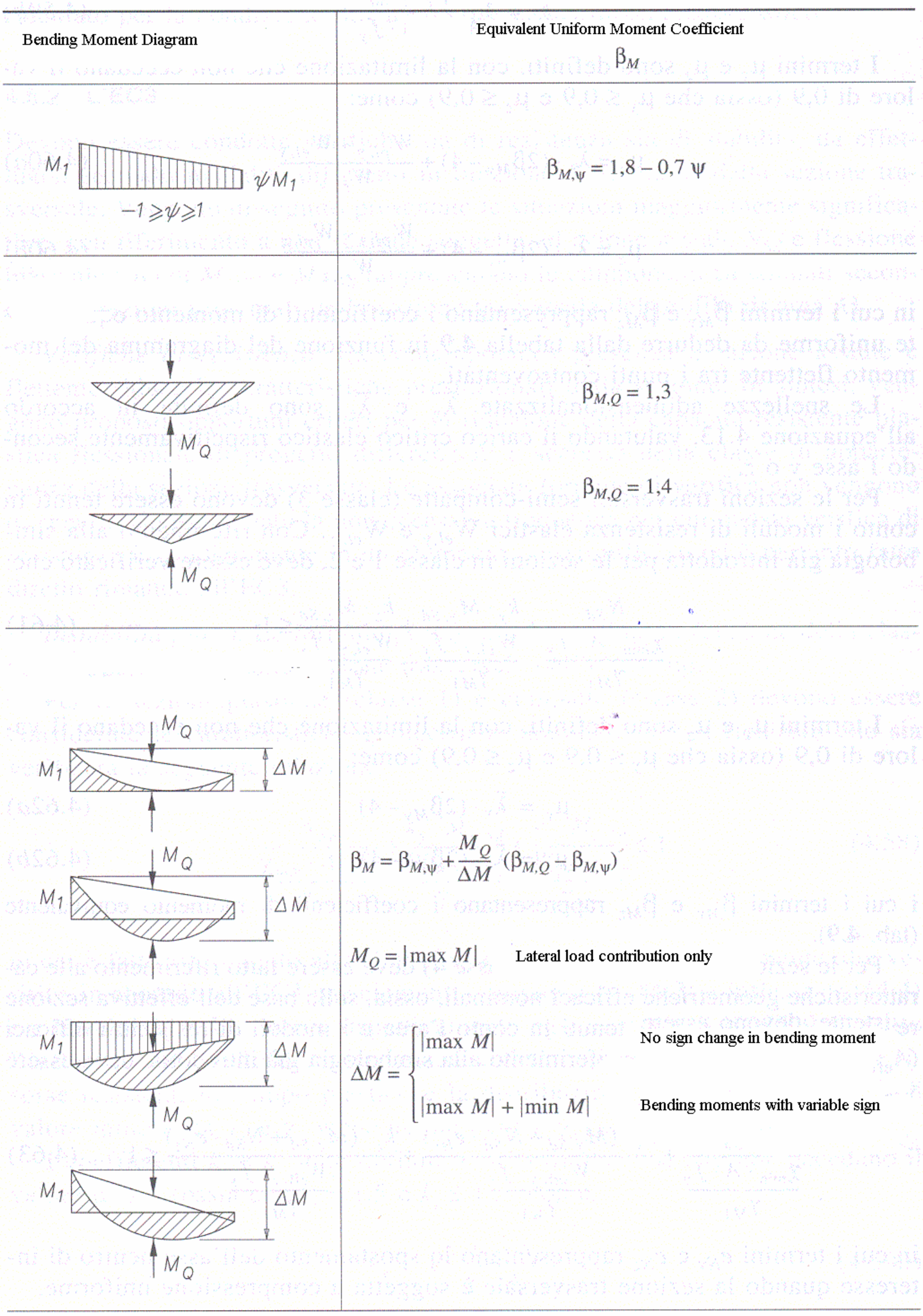


Figure 2.17: β_M factors depending on the shape of bending moment diagram.

Stability check of beam columns in class 3 can be carried out by simply substituting the plastic moduli with the elastic ones in equations from (2.64) to (2.66). Finally, for slender sections (Class 4) the relevant properties (area and strength moduli) of the effective section have to be evaluated and the bending moments need to be updated for taking into account the eccentricities $e_{N,x}$ and $e_{N,y}$ between the original centroid and the one of the effective section:

$$M_{y,Sd,eff} = M_{y,Sd} + N_{y,Sd} \cdot e_{Ny} \quad . \qquad (2.67)$$

2.8.1.3.2 EN 1-1-1993 [14] procedure.

Few formal variations has been introduced in the final version of the Eurocode accepted as EN. In particular, a general expression for the stability check of beam-columns is proposed in the following form:

$$\frac{N_{Sd}}{\frac{\chi_y N_{Rk}}{\gamma_{M1}}} + k_{yy}\frac{M_{y,Sd}+N_{Ed}e_{Ny}}{\frac{\chi_y M_{y,Rk}}{\gamma_{M1}}} + k_{yz}\frac{M_{z,Sd}+N_{Ed}e_{Nz}}{\frac{\chi_y M_{z,Rk}}{\gamma_{M1}}} \le 1$$
$$\frac{N_{Sd}}{\frac{\chi_z N_{Rk}}{\gamma_{M1}}} + k_{zy}\frac{M_{y,Sd}+N_{Ed}e_{Ny}}{\frac{\chi_y M_{y,Rk}}{\gamma_{M1}}} + k_{zz}\frac{M_{z,Sd}+N_{Ed}e_{Nz}}{\frac{\chi_y M_{z,Rk}}{\gamma_{M1}}} \le 1 \quad , \qquad (2.68)$$

provided that no lateral-torsional buckling phenomena (which will be addressed in the 4th chapter) exist.

The values of N_{Rk}, $M_{y,Rk}$ and $M_{z,Rk}$ are defined in the following general form:

$$N_{Rk} = A_i f_{ay} \ , \qquad (2.69)$$

$$M_{y,Rk} = W_y f_{ay} \qquad M_{z,Rk} = W_z f_{ay} \ . \qquad (2.70)$$

The geometrical properties reported in equations (2.69) and (2.70) can be assumed depending on the class of the transverse section as briefly summarized in Table 2.5.

Table 2.5: Values of geometrical properties depending of class section.

Class	1	2	3	4
A_i	A	A	A	A_{eff}
W_y	$W_{pl,y}$	$W_{pl,y}$	$W_{el,y}$	$W_{eff,y}$
W_z	$W_{pl,z}$	$W_{pl,z}$	$W_{el,z}$	$W_{eff,z}$

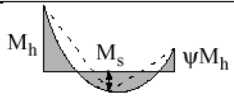
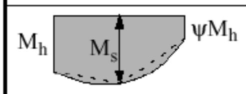
No substantial differences exist among the aspects described above. Nevertheless, as one compare equation (2.64) (and the corresponding ones for sections in class 3 or 4) with equation (2.68) can easily observe the key difference between the two approaches. In fact, four interaction factors k_{ij} (rather than two) are involved in equation (2.68) meaning that the bending contribution is different in the cases of buckling occurring either in *y* or *z* direction.

Table 2.6: Interaction factors according to Method 2 – Annex B [14].

Interaction factors	Type of sections	Design assumptions: elastic cross-sectional properties class 3, class 4	Design assumptions: plastic cross-sectional properties class 1, class 2
k_{yy}	I-sections RHS-sections	$C_{my}\left(1+0{,}6\bar{\lambda}_y\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right) \le C_{my}\left(1+0{,}6\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right)$	$C_{my}\left(1+\left(\bar{\lambda}_y-0{,}2\right)\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right) \le C_{my}\left(1+0{,}8\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right)$
k_{yz}	I-sections RHS-sections	k_{zz}	$0{,}6\,k_{zz}$
k_{zy}	I-sections RHS-sections	$0{,}8\,k_{yy}$	$0{,}6\,k_{yy}$
k_{zz}	I-sections	$C_{mz}\left(1+0{,}6\bar{\lambda}_z\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right) \le C_{mz}\left(1+0{,}6\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right)$	$C_{mz}\left(1+\left(2\bar{\lambda}_z-0{,}6\right)\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right) \le C_{mz}\left(1+1{,}4\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right)$
	RHS-sections		$C_{mz}\left(1+\left(\bar{\lambda}_z-0{,}2\right)\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right) \le C_{mz}\left(1+0{,}8\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right)$
For I- and H-sections and rectangular hollow sections under axial compression and uniaxial bending $M_{y,Ed}$ the coefficient k_{zy} may be $k_{zy}=0$.			

Two alternative approaches are reported in EC3 [14] for determining the values of such factors; they are reported in two different annexes at the same document. Only the so-called "Method 2" reported in Annex B is explicitly reported herein for the sake of brevity; its formulation for the case in which members are not susceptible of lateral-torsional buckling is summarized in Table 2.6. Finally, Table 2.7 summarized how the equivalent uniform moment factors have to be evaluated according to the mentioned Method 2.

Table 2.7: Equivalent uniform moment factors according to Method 2 – Annex B [14].

Moment diagram	range		C_{my} and C_{mz} and C_{mLT} uniform loading	C_{my} and C_{mz} and C_{mLT} concentrated load
	$-1 \le \psi \le 1$		$0,6 + 0,4\psi \ge 0,4$	$0,6 + 0,4\psi \ge 0,4$
	$0 \le \alpha_s \le 1$	$-1 \le \psi \le 1$	$0,2 + 0,8\alpha_s \ge 0,4$	$0,2 + 0,8\alpha_s \ge 0,4$
	$-1 \le \alpha_s < 0$	$0 \le \psi \le 1$	$0,1 - 0,8\alpha_s \ge 0,4$	$-0,8\alpha_s \ge 0,4$
$\alpha_s = M_s/M_h$		$-1 \le \psi < 0$	$0,1(1-\psi) - 0,8\alpha_s \ge 0,4$	$0,2(-\psi) - 0,8\alpha_s \ge 0,4$
	$0 \le \alpha_h \le 1$	$-1 \le \psi \le 1$	$0,95 + 0,05\alpha_h$	$0,90 + 0,10\alpha_h$
	$-1 \le \alpha_h < 0$	$0 \le \psi \le 1$	$0,95 + 0,05\alpha_h$	$0,90 + 0,10\alpha_h$
$\alpha_h = M_h/M_s$		$-1 \le \psi < 0$	$0,95 + 0,05\alpha_h(1+2\psi)$	$0,90 - 0,10\alpha_h(1+2\psi)$

For members with sway buckling mode the equivalent uniform moment factor should be taken $C_{my} = 0,9$ or $C_{Mz} = 0,9$ respectively.

C_{my}, C_{mz} and C_{mLT} should be obtained according to the bending moment diagram between the relevant braced points as follows:

moment factor	bending axis	points braced in direction
C_{my}	y-y	z-z
C_{mz}	z-z	y-y
C_{mLT}	y-y	y-y

2.8.2 Stability check of beam-columns according to the Italian Code

A new Code of Standards (New Technical Code, NTC in the following) has been recently issued in Italy through the D.M. 14/01/2008 which basically adopts the basic philosophy and most of the specific formulations provided by Eurocodes.

Consequently, one of the most relevant innovations with respect to the past Italian Codes consists in adopting the Limit States as the unique method for measureing structural safety and carrying out the strength and stability checks of members and strutures.

The usual equation of safety checks according to Limit States Method can be consequently introduced as the design value of stresses S_d should not be greater than the corresponding design value of Resistance R_d:

$$S_d \le R_d \ . \tag{2.71}$$

Stability check is one of the possible Ultimate Limit Verifications to be carried out on structures. Design values of stresses have to be derived by analyzing the structure under the design forces obtained by the well-known combination of permanent (self weights G_k and permanent loads G_k') and live actions Q_k:

$$F_d = \gamma_g \cdot (G_k + G_k') + \gamma_q \cdot \left[Q_{k1} + \sum_{i=2}^{n} \psi_{0i} Q_{ki} \right] \ . \tag{2.72}$$

Resistance R_d can be generally evaluated with reference to the design values of material strengths; in particular, yielding stress f_{yk} is the mechanical properties of key importance for steel structures and the design value can be obtained as a function of the characteristic one through the definition of the partial safety factors γ_m:

$$f_{ad} = \frac{f_{yk}}{\gamma_m} \ . \tag{2.73}$$

The relevant numerical values to be considered within equation (2.73) are basically the same reported in Table 2.1.

Two possible ways can be followed for evaluating the design strength R_d of members as a function of material properties f_{ad}:

- plastic analysis of cross section in class 1 and 2 could be carried out for deriving the ultimate (plastic) bending moment of members as follows:

$$M_{Rd} = W_{pl} f_{ad} \ ; \tag{2.74}$$

- conventional elastic analysis of cross section in class 3 could be even carried out for evaluating the ultimate (plastic) bending moment of members:

$$M_{Rd} = W_{el} f_{ad} \ ; \tag{2.75}$$

2.8.2.1 Stability check under axial load

Stability check is one of the key steps for steel members according to the present code and it is carried out according to the same method presented in section 2.8.1.2.

2.8.2.2 Stability check under eccentric axial load

Eccentricity hugely affects both strength and stability checks in steel members and the validity of the following relationship has to be verified for the member not to fail in buckling:

$$\frac{N_{Ed} \cdot \gamma_{M1}}{\chi_{min} \cdot f_{yk} \cdot A} + \frac{M_{yeq,Ed} \cdot \gamma_{M1}}{f_{yk} \cdot W_y \cdot \left(1 - \frac{N_{Ed}}{N_{cr,y}}\right)} + \frac{M_{zeq,Ed} \cdot \gamma_{M1}}{f_{yk} \cdot W_z \cdot \left(1 - \frac{N_{Ed}}{N_{cr,z}}\right)} \leq 1 \tag{2.76}$$

,

where bending moments M_x and M_y around the two principal axes of inertia of the section are involved through an equivalent value whose meaning will be better explained in the following; magnification factors also appear for amplifying the stress contributions of bending moments. The parameters W_y and W_z can be chosen according to the usual criterion:

- sections in class 1 or 2: $W_y=W_{pl,y}$ and $W_z=W_{pl,z}$;
- section in class 3: $W_y=W_{el,y}$ and $W_z=W_{el,z}$.

The two values of the critical loads in y and z directions can be easily evaluated as a function of the geometrical properties of the column and its cross section:

$$N_{cr,y} = \frac{\pi^2 EI_y}{L_{0,y}^{\ 2}} \ , \ N_{cr,z} = \frac{\pi^2 EI_z}{L_{0,z}^{\ 2}} \ . \tag{2.77}$$

Finally, the meaning of the equivalent bending moments M_{eq} needs to be clarified as a function of the shape of the corresponding diagram. First of all, equivalent bending moment $M_{eq}=M_0$ as an uniform bending moment diagram is considered. Secondly the following relationship can be considered among the equivalent value and the two nodal ones M_a and M_b, being $|M_a| \geq |M_b|$:

$$M_{eq} = 0.6 M_a - 0.4 M_b \geq 0.4 M_a \ . \tag{2.78}$$

Figure 2.18: Equivalent bending moment for stability check of beam-columns.

In a more general case of non linear (i.e. parabolic) shape of the bending moment diagram the equivalent bending moment can be determined as follows:

$$M_{eq} = 1.3 M_m \quad . \tag{2.79}$$

M_m being the average value of bending moment throughout the column axis and always considering the following limitation:

$$0.75 M_{max} \le M_{eq} \le M_{max} \quad . \tag{2.80}$$

Finally, Italian code also allows designers to consider the formulation reported in paragraph 2.8.1.3.2 for performing stability checks of beam columns.

2.9 Applications

Application of the above theory is proposed in the following. Some worked examples deal with the main topics covered by this section, while few unworked one are left to the reader.

2.9.1 Worked examples

Three worked examples dealing with the key topics discussed in the above sections are proposed in the following.

2.9.1.1 Euler load for a generally restrained beam-column

The first example deals with the evaluation of the Euler critical load for a generally restrained beam-column. In particular, flexible restraints are present at both ends and their flexibility is defined as follows:

$$\varepsilon_A = \frac{L_{col}}{10 EI_{col}} \quad , \tag{2.81}$$

$$\varepsilon_B = \frac{L_{col}}{5 EI_{col}} \quad . \tag{2.82}$$

being L_{col}=5.0 m the member span length and I_{col} the moment of inertia of the transverse section with reference to an axis normal to the plane of possible buckling occurrence. The beam-column is represented in Figure 2.19; since it is a non-sway member equation (2.44) or the alignment chart in Figure 2.10a has to be used for evaluating the effective length L_0 or, equivalently, the β coefficient to be adopted in equation (2.43).

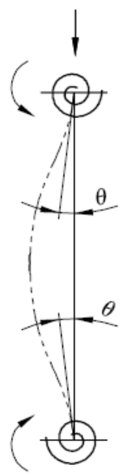


Figure 2.19: Structural scheme of the beam-column.

First of all, the values of the non-dimensional flexibilities k_A and k_B have to be determined according to their definition:

$$k_A = \frac{\varepsilon_A}{L_{col}/EI_{col}} = \frac{1}{10} = 0.10 \quad , \tag{2.83}$$

$$k_B = \frac{\varepsilon_B}{L_{col}/EI_{col}} = \frac{1}{5} = 0.20 \ .$$

As a matter of principle, the corresponding value of the β coefficient is within the range [0.5, 1.0], being the column a non-sway member. The equation (2.44) can be applied for determining its value:

$$\beta = 0.5 \cdot \sqrt{\left(1+\frac{0.10}{0.45+0.10}\right)\cdot\left(1+\frac{0.20}{0.45+0.20}\right)} = 0.622 \ . \tag{2.84}$$

The above value is rather conservative with respect to the one which could be derived by the alignment chart in figure Figure 2.10a, as desirable for an approximate formula.

2.9.1.2 Stability check of an axially loaded beam-column

Let us consider a member in compression whose transverse section is realized by a profile HE 200 B made out of steel S235. The overall span length of the member is 7.5 m and displacement in the direction perpendicular to the web plane constraints the displacements in that direction (Figure 2.20).

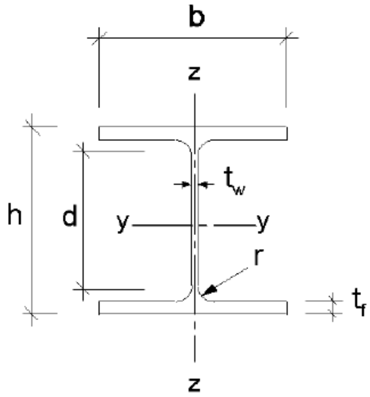


Figure 2.20: Beam-column under axial force.

The transverse section of the beam is characterized by the following geometrical parameters:

- depth	h	200 mm;
- width	b	200 mm;
- flange thickness	t_f	15 mm;
- web thickness	t_w	9 mm;
- radius	r	18 mm;
- area	A	7810 mm^2;
- Moment of inertia with respect to the strong axis	I_y	5696 10^4 mm^4;
- Moment of inertia with respect to the weak axis	I_z	2003 10^4 mm^4.

For the sake of brevity the exercise will be only solved with reference to the EC3 provisions for stability check.

Step #1: classification of the transverse section:

Since the adopted steel grade is f_y=235 MPa the value ε=1 can be assumed for the parameter mentioned in Table 2.2and Table 2.3. The following values of the length-to-thickness ratios can be evaluated for flange and web:

- flange c/t_f=(200/2)/15=6.7≤10 Class 1;
- web d/t_w=(200-2 ·15-2 ·18)/9=14.9≤33 Class 1

Finally, the profile HE200B made out of steel S235 is in class 1 if loaded in compression.

Step #2.1: evaluating the (elastic) Euler load for buckling along the strong axis:

Since hinged restraints can be recognized for both ends and no further constraints control displacements in the mentioned direction, the effective length $L_{0,y}$ is equal to the nominal one (L=7500 mm) and the Euler load can be easily derived:

$$N_{cr,y} = \frac{\pi^2 EI_y}{L_{0,y}^{\ 2}} = \frac{\pi^2 210000 \cdot 56960000}{7500^2} = 2098.78 \ kN \ . \tag{2.85}$$

<u>Step #2.2: evaluating relative (non-dimensional) slenderness along the strong axis:</u>

$$\bar{\lambda}_y = \sqrt{\frac{\beta_A A f_{ay}}{N_{cr,y}}} = \sqrt{\frac{1 \cdot 7810 \cdot 235}{2098.78 \cdot 10^3}} = 0.935 \ . \tag{2.86}$$

<u>Step #2.3: determination of the reduction factor χ_y:</u>

According to Figure 2.15 the profile follows the *curve b* and, consequently, the following value of the reduction factor χ_y can be evaluated:

$$\Phi_y = 0.5 \cdot \left[1 + \alpha \cdot \left(\bar{\lambda}_y - 0.2\right) + \bar{\lambda}_y^{\ 2}\right] = 0.5 \cdot \left[1 + 0.34 \cdot (0.935 - 0.2) + 0.935^2\right] = 1.062 \ , \tag{2.87}$$

and

$$\chi_y = \frac{1}{\Phi_y + \sqrt{\Phi_y^{\ 2} - \bar{\lambda}_y^{\ 2}}} = \frac{1}{1.062 + \sqrt{1.062^2 - 0.935^2}} = 0.6387 \ . \tag{2.88}$$

<u>Step #3.1: evaluating the (elastic) Euler load for buckling along the weak axis:</u>

Since transverse displacements are constrained at mid-span, the effective length $L_{0,z}$ is one half of the nominal one (L=7500 mm) and the Euler load can be easily derived:

$$N_{cr,z} = \frac{\pi^2 EI_z}{L_{0,z}^{\ 2}} = \frac{\pi^2 210000 \cdot 20030000}{3250^2} = 2952.10 \ kN \ . \tag{2.89}$$

<u>Step #3.2: evaluating relative (non-dimensional) slenderness along the weak axis:</u>

$$\bar{\lambda}_z = \sqrt{\frac{\beta_A A f_{ay}}{N_{cr,z}}} = \sqrt{\frac{1 \cdot 7810 \cdot 235}{2952.1 \cdot 10^3}} = 0.788 \ . \tag{2.90}$$

<u>Step #3.3: determination of the reduction factor χ_y:</u>

According to Figure 2.15 the profile follows the *curve c* and, consequently, the following value of the reduction factor χ_z can be evaluated:

$$\Phi_z = 0.5 \cdot \left[1 + \alpha \cdot \left(\bar{\lambda}_z - 0.2\right) + \bar{\lambda}_z^{\ 2}\right] = 0.5 \cdot \left[1 + 0.49 \cdot (0.788 - 0.2) + 0.788^2\right] = 0.955 \ , \tag{2.91}$$

and

$$\chi_z = \frac{1}{\Phi_z + \sqrt{\Phi_z^{\ 2} - \bar{\lambda}_z^{\ 2}}} = \frac{1}{0.955 + \sqrt{0.955^2 - 0.788^2}} = 0.6695 \ . \tag{2.92}$$

<u>Step #4: evaluating the ultimate axial load capacity:</u>

The minimum value of the reduction factor evaluated along the two directions has to be considered for determining the ultimate bearing capacity of the member.

$$N_{b,Rd} = \chi_{\min} \beta_A A \frac{f_{ay}}{\gamma_{M1}} = 0.6387 \cdot 7810 \cdot \frac{235}{1.05} = 1116.4 \ kN \ . \tag{2.93}$$

2.9.1.3 Stability check of an eccentrically loaded beam-column

The beam-column represented in Figure 2.21 has the same transverse section described in the previous example. Transverse displacements are constrained at the top of the column since the two following values of the β coefficient can be assumed:

- buckling in the strong direction (perpendicular to *y* axis) β_y=2.0;
- buckling in the weak direction (perpendicular to *z* axis) β_z=1.0;

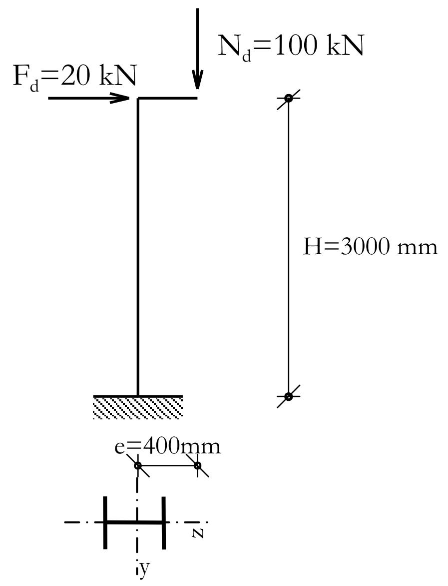


Figure 2.21: Beam-column under eccentric axial force.

Step #1: classification of the transverse section:

According to the findings of the previous exercise, the profile HE200B made out of steel S235 is in class 1 if loaded in compression.

Step #2.1: evaluating the (elastic) Euler load for buckling along the strong axis:

Since hinged restraints can be recognized for both ends and no further constraints control displacements in the mentioned direction, the effective length $L_{0,y}$ is equal to the nominal one (L=7500 mm) and the Euler load can be easily derived:

$$N_{cr,y} = \frac{\pi^2 EI_y}{L_{0,y}^2} = \frac{\pi^2 210000 \cdot 56960000}{6000^2} = 3279.15\ kN \ . \tag{2.94}$$

Step #2.2: evaluating relative (non-dimensional) slenderness along the strong axis:

$$\bar{\lambda}_y = \sqrt{\frac{\beta_A A f_{ay}}{N_{cr,y}}} = \sqrt{\frac{1 \cdot 7810 \cdot 235}{3279.15 \cdot 10^3}} = 0.748 \ . \tag{2.95}$$

Step #2.3: determination of the reduction factor χ_y:

According to Figure 2.15 the profile follows the *curve b* and, consequently, the following value of the reduction factor χ_y can be evaluated:

$$\Phi_y = 0.5 \cdot \left[1 + \alpha \cdot \left(\bar{\lambda}_y - 0.2\right) + \bar{\lambda}_y^2\right] = 0.5 \cdot \left[1 + 0.34 \cdot \left(0.748 - 0.2\right) + 0.748^2\right] = 0.873 \ , \tag{2.96}$$

and

$$\chi_y = \frac{1}{\Phi_y + \sqrt{\Phi_y^2 - \bar{\lambda}_y^2}} = \frac{1}{0.873 + \sqrt{0.873^2 - 0.748^2}} = 0.7559 \ . \tag{2.97}$$

Step #3: evaluation of the relevant interaction factors:

Since no bending moment is applied around the z-axis the only interaction factor to be determined for applying the first one of the two equations (2.78) is the term k_{yy}. Linear variation can be observed for the bending moment around y-axis, which varies from the two following values:

- $M_{y,Sd,top} = N_d \cdot e = 100 \cdot 0.40 = 40\ kNm$;
- $M_{y,Sd,bottom} = N_d \cdot e + F_d \cdot H = 100 \cdot 0.40 + 20 \cdot 3.0 = 100\ kNm$;

Consequently, the maximum moment to be considered in equation (2.78) is $M_{y,Sd}$= 100 kNm and, according to Table 2.7 the following value can be assumed for the equivalent uniform moment factor $C_{m,y}$:

$$\psi = \frac{M_{y,Sd,top}}{M_{y,Sd,bottom}} = \frac{40}{100} = 0.40 \implies C_{m,y} = 0.6 + 0.4\psi = 0.76 \ . \qquad (2.98)$$

Finally, the value of the interaction factor k_{yy} can be derived according to the formulae reported in the first row of Table 2.6, provided that $\bar{\lambda}_y < 1.0$:

$$k_{yy} = C_{my} \cdot \left[1 + \left(\bar{\lambda}_y - 0.2 \right) \cdot \frac{N_{Sd}}{\chi_y A f_{ay} / \gamma_{M1}} \right] = 0.76 \cdot \left[1 + \left(0.748 - 0.2 \right) \cdot \frac{100000}{0.748 \cdot 7810 \cdot 235 / 1.05} \right] = 1.042 \ . \qquad (2.99)$$

Step #4.1: evaluating the (elastic) Euler load for buckling along the weak axis:

Since transverse displacements are constrained at mid-span, the effective length $L_{0,z}$ is one half of the nominal one (H=6000 mm) and the Euler load can be easily derived:

$$N_{cr,z} = \frac{\pi^2 E I_z}{L_{0,z}^{\ 2}} = \frac{\pi^2 210000 \cdot 20030000}{3000^2} = 4612.45 \ kN \ . \qquad (2.100)$$

Step #4.2: evaluating relative (non-dimensional) slenderness along the weak axis:

$$\bar{\lambda}_z = \sqrt{\frac{\beta_A A f_{ay}}{N_{cr,z}}} = \sqrt{\frac{1 \cdot 7810 \cdot 235}{2952.1 \cdot 10^3}} = 0.631 \ . \qquad (2.101)$$

Step #4.3: determination of the reduction factor χ_y:

According to Figure 2.15 the profile follows the *curve c* and, consequently, the following value of the reduction factor χ_z can be evaluated:

$$\Phi_z = 0.5 \cdot \left[1 + \alpha \cdot \left(\bar{\lambda}_z - 0.2 \right) + \bar{\lambda}_z^{\ 2} \right] = 0.5 \cdot \left[1 + 0.49 \cdot \left(0.631 - 0.2 \right) + 0.631^2 \right] = 0.805 \ , \qquad (2.102)$$

and

$$\chi_z = \frac{1}{\Phi_z + \sqrt{\Phi_z^{\ 2} - \bar{\lambda}_z^{\ 2}}} = \frac{1}{0.805 + \sqrt{0.805^2 - 0.631^2}} = 0.7664 \ . \qquad (2.103)$$

Step #5: evaluation of the relevant interaction factors:

Since no bending moment is applied around the z-axis the only interaction factor to be determined for applying the second one of the two equations (2.78) is the term k_{zy}. An easy relationship is stated for determining this factor as a function of k_{yy} as follows:

$$k_{zy} = 0.6 k_{yy} = 0.625 \ . \qquad (2.104)$$

Step #6: final stability check:

The two equations (2.78) can be finally applied for checking the given structure against global buckling:

$$\begin{aligned} &\frac{100000}{\frac{0.7559 \cdot 7810 \cdot 235}{1.05}} + 1.042 \cdot \frac{100000000}{\frac{0.7559 \cdot 56960000 \cdot 235}{1.05}} = 0.076 + 0.011 = 0.087 \le 1 \\ &\frac{100000}{\frac{0.7664 \cdot 7810 \cdot 235}{1.05}} + 0.625 \cdot \frac{100000000}{\frac{0.7664 \cdot 20030000 \cdot 235}{1.05}} = 0.075 + 0.018 = 0.093 \le 1 \end{aligned} \ . \qquad (2.105)$$

2.9.2 Unworked examples

The following exercises are left to the readers:

1) for the same beam-column reported in paragraph 2.9.1.1, evaluate the β coefficient in the case of sway member. Compare the results obtained by the simplified formula and the alignment chart;
2) for the same beam-column described in paragraph 2.9.1.2, evaluate the ultimate load bearing capacity according to the Italian Code;

3) for the same beam-column described in paragraph 2.9.1.2, evaluate the ultimate load bearing capacity considering a steel grade s355;
4) for the same beam-column described in paragraph 2.9.1.3, evaluate the maximum lateral load $F_{Sd,max}$ with reference to the global stability check of the structure;
5) for the same beam-column described in paragraph 2.9.1.3, perform the stability check according to the Italian code.