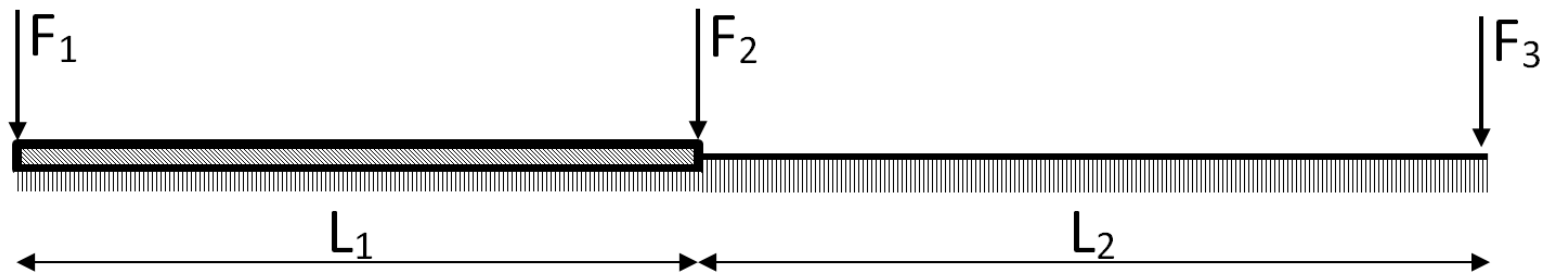


---

## Schema strutturale



---

## Soluzione generale

Si introducono le espressioni generali dei campi di spostamenti relativi ai due tratti del sistema di travi rappresentato sopra.

### ■ Tratto 1

```
In[1]:= Costanti1 = {A0, A1};  
Base1 = {1, x1};  
w1 = Costanti1.Base1
```

```
Out[3]= A0 + A1 x1
```

## ■ Tratto 2

```
In[4]:= Costanti2 = {B1, B2, B3, B4};
      Base2 = {Exp[-α x2] Sin[α x2], Exp[-α x2] Cos[α x2], Exp[α x2] Sin[α x2], Exp[α x2] Cos[α x2]};
      w2 = Costanti2.Base2
```

```
Out[6]= B2 e-x2 α Cos[x2 α] + B4 ex2 α Cos[x2 α] + B1 e-x2 α Sin[x2 α] + B3 ex2 α Sin[x2 α]
```

---

## Condizioni globali di equilibrio, condizioni al contorno e di accoppiamento

### ■ Equazioni globali di equilibrio sul tratto 1

Agli estremi del tratto infinitamente rigido non è possibile esprimere le caratteristiche della sollecitazione (Momento Flettente e Taglio) in funzione delle derivate seconda e terza della linea elastica, in quanto non è definito un numero reale che rappresenti la rigidezza flessionale "EI" di quel tratto. Di conseguenza, le derivate seconda e terza sarebbero nulle, mentre le caratteristiche della sollecitazione non lo sono necessariamente: esse assumono valore reale (non nullo in ragione delle forze nodali effettivamente applicate) e, matematicamente, sono il risultato di un prodotto di una quantità infinitesima (le derivate di ordine superiore di  $w_1$ ) e una infinita (il termine "EI").

Pertanto sul tratto 1 vanno scritte equazioni globali di equilibrio che tengano conto delle azioni e delle reazioni applicate sulla stessa, per effetto della presenza del suolo alla Winkler e del tratto 2.

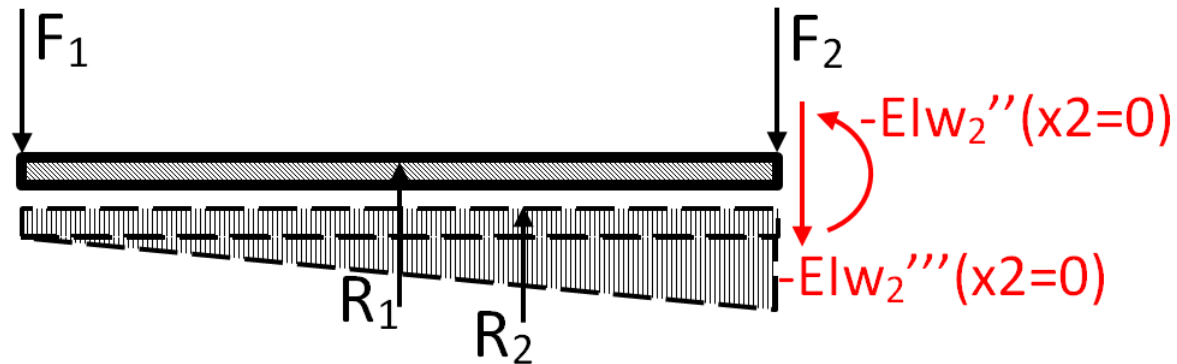


Figura 2: diagramma di corpo libero: azioni e reazioni applicate sul tratto 1.

### ■ Equilibrio globale del tratto 1

`In[7]:= R1 = k (w1 /. x1 -> 0) L1`

`R2 = k ((w1 /. x1 -> L1) - (w1 /. x1 -> 0)) L1 / 2`

`Out[7]= A0 k L1`

`Out[8]=  $\frac{1}{2}$  A1 k L12`

`In[9]:= Eq1 = (-EI D[w2, {x2, 3}] /. x2 -> 0) + F1 + F2 - R1 - R2`

`Eq2 = (-EI D[w2, {x2, 2}] /. x2 -> 0) + F1 L1 - R1 L1 / 2 - R2 L1 / 3`

`Out[9]= F1 + F2 - A0 k L1 -  $\frac{1}{2}$  A1 k L12 - EI (2 B1  $\alpha^3$  + 2 B2  $\alpha^3$  + 2 B3  $\alpha^3$  - 2 B4  $\alpha^3$ )`

`Out[10]= F1 L1 -  $\frac{1}{2}$  A0 k L12 -  $\frac{1}{6}$  A1 k L13 - EI (-2 B1  $\alpha^2$  + 2 B3  $\alpha^2$ )`

## ■ Congruenza tra i tratti 1 e 2

```
In[11]:= Eq3 = (w2 /. x2 -> 0) - (w1 /. x1 -> L1)
Eq4 = (-D[w2, x2] /. x2 -> 0) - (-D[w1, x1] /. x1 -> L1)
```

```
Out[11]= -A0 + B2 + B4 - A1 L1
```

```
Out[12]= A1 - B1 α + B2 α - B3 α - B4 α
```

## ■ Condizioni al contorno (statiche) all'estremo del tratto 2 ( $x_2=L_2$ )

```
In[13]:= Eq5 = -EI D[w2, {x2, 2}] /. x2 -> L2
Eq6 = (-EI D[w2, {x2, 3}] /. x2 -> L2) - F3
```

```
Out[13]= -EI (-2 B1 e-L2 α α2 Cos[L2 α] + 2 B3 eL2 α α2 Cos[L2 α] + 2 B2 e-L2 α α2 Sin[L2 α] - 2 B4 eL2 α α2 Sin[L2 α])
```

```
Out[14]= -F3 - EI (2 B1 e-L2 α α3 Cos[L2 α] + 2 B2 e-L2 α α3 Cos[L2 α] + 2 B3 eL2 α α3 Cos[L2 α] -
2 B4 eL2 α α3 Cos[L2 α] + 2 B1 e-L2 α α3 Sin[L2 α] - 2 B2 e-L2 α α3 Sin[L2 α] - 2 B3 eL2 α α3 Sin[L2 α] - 2 B4 eL2 α α3 Sin[L2 α])
```

## ■ Sistema risolutivo

```
In[15]:= Costanti = Join[Costanti1, Costanti2]
BC = {Eq1, Eq2, Eq3, Eq4, Eq5, Eq6}
```

```
Out[15]= {A0, A1, B1, B2, B3, B4}
```

```
Out[16]= {F1 + F2 - A0 k L1 -  $\frac{1}{2}$  A1 k L12 - EI (2 B1 α3 + 2 B2 α3 + 2 B3 α3 - 2 B4 α3), F1 L1 -  $\frac{1}{2}$  A0 k L12 -  $\frac{1}{6}$  A1 k L13 - EI (-2 B1 α2 + 2 B3 α2), -A0 + B2 + B4 - A1 L1,
A1 - B1 α + B2 α - B3 α - B4 α, -EI (-2 B1 e-L2 α α2 Cos[L2 α] + 2 B3 eL2 α α2 Cos[L2 α] + 2 B2 e-L2 α α2 Sin[L2 α] - 2 B4 eL2 α α2 Sin[L2 α]),
-F3 - EI (2 B1 e-L2 α α3 Cos[L2 α] + 2 B2 e-L2 α α3 Cos[L2 α] + 2 B3 eL2 α α3 Cos[L2 α] - 2 B4 eL2 α α3 Cos[L2 α] +
2 B1 e-L2 α α3 Sin[L2 α] - 2 B2 e-L2 α α3 Sin[L2 α] - 2 B3 eL2 α α3 Sin[L2 α] - 2 B4 eL2 α α3 Sin[L2 α]) }
```

```
In[17]:= CostantiSol = FullSimplify[(Solve[BC == 0, Costanti] // Flatten)]
```

Out[17]= {A0 →

$$\begin{aligned}
& - (2 (2 (-2 F1 + F2) k L1^3 \alpha + L1 \alpha (- (2 F1 - F2) k L1^2 + 12 EI (2 F1 + F2) \alpha^2) \cos[2 L2 \alpha] + L1 \alpha (- (2 F1 - F2) k L1^2 - 12 EI (2 F1 + F2) \alpha^2) \cosh[2 L2 \alpha] \\
& \quad + 2 F3 \cosh[L2 \alpha] (2 k L1^3 \alpha \cos[L2 \alpha] + 3 (k L1^2 + 4 EI \alpha^2) \sin[L2 \alpha]) + 6 (F3 ((k L1^2 - 4 EI \alpha^2) \cos[L2 \alpha] + 8 EI L1 \alpha^3 \sin[L2 \alpha]) \\
& \quad \sinh[L2 \alpha] + 2 EI \alpha^2 ((F1 + F2 - 2 F1 L1^2 \alpha^2) \sin[2 L2 \alpha] - (F1 + F2 + 2 F1 L1^2 \alpha^2) \sinh[2 L2 \alpha]))) / \\
& (\alpha (2 k^2 L1^4 - 96 EI^2 \alpha^4 + (k^2 L1^4 - 24 EI k L1^2 \alpha^2 + 48 EI^2 \alpha^4) \cos[2 L2 \alpha] + (k^2 L1^4 + 24 EI k L1^2 \alpha^2 + 48 EI^2 \alpha^4) \cosh[2 L2 \alpha] + \\
& \quad 8 EI k L1 \alpha ((-3 + 2 L1^2 \alpha^2) \sin[2 L2 \alpha] + (3 + 2 L1^2 \alpha^2) \sinh[2 L2 \alpha]))) , \\
A1 \rightarrow & (6 (2 (-F1 + F2) k L1^2 \alpha + \alpha (- (F1 - F2) k L1^2 + 4 EI (F1 + F2) \alpha^2) \cos[2 L2 \alpha] - \alpha ((F1 - F2) k L1^2 + 4 EI (F1 + F2) \alpha^2) \cosh[2 L2 \alpha] + \\
& \quad 4 F3 k L1 \cosh[L2 \alpha] (L1 \alpha \cos[L2 \alpha] + \sin[L2 \alpha]) - 8 EI F1 L1 \alpha^4 \sin[2 L2 \alpha] + \\
& \quad 4 F3 (k L1 \cos[L2 \alpha] + 4 EI \alpha^3 \sin[L2 \alpha]) \sinh[L2 \alpha] - 8 EI F1 L1 \alpha^4 \sinh[2 L2 \alpha])) / \\
& (\alpha (2 k^2 L1^4 - 96 EI^2 \alpha^4 + (k^2 L1^4 - 24 EI k L1^2 \alpha^2 + 48 EI^2 \alpha^4) \cos[2 L2 \alpha] + (k^2 L1^4 + 24 EI k L1^2 \alpha^2 + 48 EI^2 \alpha^4) \cosh[2 L2 \alpha] + \\
& \quad 8 EI k L1 \alpha ((-3 + 2 L1^2 \alpha^2) \sin[2 L2 \alpha] + (3 + 2 L1^2 \alpha^2) \sinh[2 L2 \alpha]))) , \\
B1 \rightarrow & (-e^{L2 \alpha} F3 k L1^2 (k L1^2 - 12 EI \alpha^2) \cos[L2 \alpha] + 2 EI \alpha^2 (-3 (F1 - F2) k L1^2 - 12 EI (F1 + F2) \alpha^2 + 24 EI F1 L1 \alpha^3 - \\
& \quad e^{2 L2 \alpha} L1 (3 (F1 - F2) k L1 + (F1 - 2 F2) k L1^2 \alpha + 12 EI F1 \alpha^3) + \alpha ((F1 - 2 F2) k L1^3 + 12 EI (F1 + F2) \alpha - 12 EI F1 L1 \alpha^2) \cos[2 L2 \alpha]) - \\
& \quad 2 EI L1 \alpha^2 (F2 k L1 (3 - 2 L1 \alpha) + F1 (12 EI \alpha^3 + k L1 (-3 + L1 \alpha))) \sin[2 L2 \alpha] + \\
& \quad F3 \sin[L2 \alpha] (-k L1 (k L1^3 + 12 EI \alpha (-2 + L1 \alpha)) \cosh[L2 \alpha] + 4 EI \alpha^2 (12 EI \alpha^2 + k L1^2 (3 - 4 L1 \alpha)) \sinh[L2 \alpha])) / \\
& (2 EI \alpha^3 (2 k^2 L1^4 - 96 EI^2 \alpha^4 + (k^2 L1^4 - 24 EI k L1^2 \alpha^2 + 48 EI^2 \alpha^4) \cos[2 L2 \alpha] + (k^2 L1^4 + 24 EI k L1^2 \alpha^2 + 48 EI^2 \alpha^4) \cosh[2 L2 \alpha] + \\
& \quad 8 EI k L1 \alpha ((-3 + 2 L1^2 \alpha^2) \sin[2 L2 \alpha] + (3 + 2 L1^2 \alpha^2) \sinh[2 L2 \alpha]))) , \\
B2 \rightarrow & (2 EI \alpha^2 (F2 (12 (-1 + e^{2 L2 \alpha}) EI \alpha^2 + k L1^2 (-3 + 2 (2 + e^{2 L2 \alpha}) L1 \alpha)) + F1 (-k L1^2 (-3 + (2 + e^{2 L2 \alpha}) L1 \alpha) + 12 EI \alpha^2 (-1 + e^{2 L2 \alpha} (1 + L1 \alpha))) - \\
& \quad L1 (F2 k L1 (3 - 2 L1 \alpha) + F1 (12 EI \alpha^3 + k L1 (-3 + L1 \alpha))) \cos[2 L2 \alpha] + 6 e^{L2 \alpha} F3 (k L1^2 - 4 EI \alpha^2) \sin[L2 \alpha] + \\
& \quad \alpha (- (F1 - 2 F2) k L1^3 - 12 EI (F1 + F2) \alpha + 12 EI F1 L1 \alpha^2) \sin[2 L2 \alpha]) - \\
& \quad F3 \cos[L2 \alpha] (k L1^2 (k L1^2 + 4 EI \alpha^2 (3 - 4 L1 \alpha)) \cosh[L2 \alpha] - 12 EI \alpha (4 EI \alpha^3 + k L1 (-2 + L1 \alpha)) \sinh[L2 \alpha])) / \\
& (2 EI \alpha^3 (2 k^2 L1^4 - 96 EI^2 \alpha^4 + (k^2 L1^4 - 24 EI k L1^2 \alpha^2 + 48 EI^2 \alpha^4) \cos[2 L2 \alpha] + (k^2 L1^4 + 24 EI k L1^2 \alpha^2 + 48 EI^2 \alpha^4) \cosh[2 L2 \alpha] + \\
& \quad 8 EI k L1 \alpha ((-3 + 2 L1^2 \alpha^2) \sin[2 L2 \alpha] + (3 + 2 L1^2 \alpha^2) \sinh[2 L2 \alpha]))) , \\
B3 \rightarrow & (e^{-2 L2 \alpha} (4 EI \alpha^2 (-3 e^{2 L2 \alpha} ((F1 - F2) k L1^2 + 4 EI (F1 + F2) \alpha^2 + 8 EI F1 L1 \alpha^3) + L1 (F2 k L1 (3 - 2 L1 \alpha) + F1 (12 EI \alpha^3 + k L1 (-3 + L1 \alpha)))) + \\
& \quad e^{L2 \alpha} (-2 F3 k L1^2 (k L1^2 - 12 EI \alpha^2) \cos[L2 \alpha] + 4 e^{L2 \alpha} EI \alpha^3 (- (F1 - 2 F2) k L1^3 + 12 EI (F1 + F2) \alpha + 12 EI F1 L1 \alpha^2) \cos[2 L2 \alpha] + \\
& \quad F3 ((1 + e^{2 L2 \alpha}) k^2 L1^4 + 48 (-1 + e^{2 L2 \alpha}) EI^2 \alpha^4 + 8 EI k L1 \alpha (3 - 2 L1^2 \alpha^2 + e^{2 L2 \alpha} (3 + L1 \alpha (3 + 2 L1 \alpha)))) \sin[L2 \alpha] - \\
& \quad 4 e^{L2 \alpha} EI L1 \alpha^2 (3 (F1 - F2) k L1 + (F1 - 2 F2) k L1^2 \alpha + 12 EI F1 \alpha^3) \sin[2 L2 \alpha])) / \\
& (4 EI \alpha^3 (2 k^2 L1^4 - 96 EI^2 \alpha^4 + (k^2 L1^4 - 24 EI k L1^2 \alpha^2 + 48 EI^2 \alpha^4) \cos[2 L2 \alpha] + (k^2 L1^4 + 24 EI k L1^2 \alpha^2 + 48 EI^2 \alpha^4) \cosh[2 L2 \alpha] + \\
& \quad 8 EI k L1 \alpha ((-3 + 2 L1^2 \alpha^2) \sin[2 L2 \alpha] + (3 + 2 L1^2 \alpha^2) \sinh[2 L2 \alpha]))) , \\
B4 \rightarrow & (e^{-2 L2 \alpha} (4 EI \alpha^2 (\alpha (- (F1 - 2 F2) k L1^3 - 12 EI (F1 + F2) \alpha + 12 EI F1 L1 \alpha^2) - \\
& \quad e^{2 L2 \alpha} L1 (3 (F1 - F2) k L1 + (F1 - 2 F2) k L1^2 \alpha + 12 EI F1 \alpha^3) \cos[2 L2 \alpha] + 6 e^{L2 \alpha} F3 (k L1^2 - 4 EI \alpha^2) \sin[L2 \alpha] + e^{2 L2 \alpha} \\
& \quad (-3 (F1 - F2) k L1^2 - 2 (F1 - 2 F2) k L1^3 \alpha + 12 EI (F1 + F2) \alpha^2 - \alpha (- (F1 - 2 F2) k L1^3 + 12 EI (F1 + F2) \alpha + 12 EI F1 L1 \alpha^2) \sin[2 L2 \alpha])) + \\
& \quad 2 e^{2 L2 \alpha} F3 \cos[L2 \alpha] (k L1^2 (k L1^2 + 4 EI \alpha^2 (3 + 4 L1 \alpha)) \cosh[L2 \alpha] + 12 EI \alpha (4 EI \alpha^3 + k L1 (2 + L1 \alpha)) \sinh[L2 \alpha])) / \\
& (4 EI \alpha^3 (2 k^2 L1^4 - 96 EI^2 \alpha^4 + (k^2 L1^4 - 24 EI k L1^2 \alpha^2 + 48 EI^2 \alpha^4) \cos[2 L2 \alpha] + (k^2 L1^4 + 24 EI k L1^2 \alpha^2 + 48 EI^2 \alpha^4) \cosh[2 L2 \alpha] + \\
& \quad 8 EI k L1 \alpha ((-3 + 2 L1^2 \alpha^2) \sin[2 L2 \alpha] + (3 + 2 L1^2 \alpha^2) \sinh[2 L2 \alpha]))) }
\end{aligned}$$

## Linea elastica

```
In[18]:= w1sol = w1 /. CostantiSol
w2sol = w2 /. CostantiSol
```

```
Out[18]= (6 x1 (2 (-F1 + F2) k L1^2 α + α (- (F1 - F2) k L1^2 + 4 EI (F1 + F2) α^2) Cos[2 L2 α] -
α ((F1 - F2) k L1^2 + 4 EI (F1 + F2) α^2) Cosh[2 L2 α] + 4 F3 k L1 Cosh[L2 α] (L1 α Cos[L2 α] + Sin[L2 α]) -
8 EI F1 L1 α^4 Sin[2 L2 α] + 4 F3 (k L1 Cos[L2 α] + 4 EI α^3 Sin[L2 α]) Sinh[L2 α] - 8 EI F1 L1 α^4 Sinh[2 L2 α])) /
(α (2 k^2 L1^4 - 96 EI^2 α^4 + (k^2 L1^4 - 24 EI k L1^2 α^2 + 48 EI^2 α^4) Cos[2 L2 α] + (k^2 L1^4 + 24 EI k L1^2 α^2 + 48 EI^2 α^4) Cosh[2 L2 α] +
8 EI k L1 α ((-3 + 2 L1^2 α^2) Sin[2 L2 α] + (3 + 2 L1^2 α^2) Sinh[2 L2 α]))) -
(2 (2 (-2 F1 + F2) k L1^3 α + L1 α (- (2 F1 - F2) k L1^2 + 12 EI (2 F1 + F2) α^2) Cos[2 L2 α] + L1 α (- (2 F1 - F2) k L1^2 - 12 EI (2 F1 + F2) α^2) Cosh[2 L2 α] +
2 F3 Cosh[L2 α] (2 k L1^3 α Cos[L2 α] + 3 (k L1^2 + 4 EI α^2) Sin[L2 α]) + 6 (F3 ((k L1^2 - 4 EI α^2) Cos[L2 α] + 8 EI L1 α^3 Sin[L2 α]) Sinh[L2 α] +
2 EI α^2 ((F1 + F2 - 2 F1 L1^2 α^2) Sin[2 L2 α] - (F1 + F2 + 2 F1 L1^2 α^2) Sinh[2 L2 α]))) /
(α (2 k^2 L1^4 - 96 EI^2 α^4 + (k^2 L1^4 - 24 EI k L1^2 α^2 + 48 EI^2 α^4) Cos[2 L2 α] + (k^2 L1^4 + 24 EI k L1^2 α^2 + 48 EI^2 α^4) Cosh[2 L2 α] +
8 EI k L1 α ((-3 + 2 L1^2 α^2) Sin[2 L2 α] + (3 + 2 L1^2 α^2) Sinh[2 L2 α])))
```

```

Out[19]= (e-L2 α+x2 α (4 EI α2 (-3 e2L2 α ((F1 - F2) k L12 + 4 EI (F1 + F2) α2 + 8 EI F1 L1 α3) + L1 (F2 k L1 (3 - 2 L1 α) + F1 (12 EI α3 + k L1 (-3 + L1 α)))) +
eL2 α (-2 F3 k L12 (k L12 - 12 EI α2) Cos[L2 α] + 4 eL2 α EI α3 (- (F1 - 2 F2) k L13 + 12 EI (F1 + F2) α + 12 EI F1 L1 α2) Cos[2 L2 α] +
F3 ((1 + e2L2 α) k2 L14 + 48 (-1 + e2L2 α) EI2 α4 + 8 EI k L1 α (3 - 2 L12 α2 + e2L2 α (3 + L1 α (3 + 2 L1 α)))) Sin[L2 α] -
4 eL2 α EI L1 α2 (3 (F1 - F2) k L1 + (F1 - 2 F2) k L12 α + 12 EI F1 α3) Sin[2 L2 α])) Sin[x2 α]) /
(4 EI α3 (2 k2 L14 - 96 EI2 α4 + (k2 L14 - 24 EI k L12 α2 + 48 EI2 α4) Cos[2 L2 α] + (k2 L14 + 24 EI k L12 α2 + 48 EI2 α4) Cosh[2 L2 α] +
8 EI k L1 α ((-3 + 2 L12 α2) Sin[2 L2 α] + (3 + 2 L12 α2) Sinh[2 L2 α])) +
(e-x2 α Sin[x2 α] (-eL2 α F3 k L12 (k L12 - 12 EI α2) Cos[L2 α] + 2 EI α2 (-3 (F1 - F2) k L12 - 12 EI (F1 + F2) α2 + 24 EI F1 L1 α3 -
e2L2 α L1 (3 (F1 - F2) k L1 + (F1 - 2 F2) k L12 α + 12 EI F1 α3) + α ((F1 - 2 F2) k L13 + 12 EI (F1 + F2) α - 12 EI F1 L1 α2) Cos[2 L2 α]) -
2 EI L1 α2 (F2 k L1 (3 - 2 L1 α) + F1 (12 EI α3 + k L1 (-3 + L1 α))) Sin[2 L2 α] +
F3 Sin[L2 α] (-k L1 (k L13 + 12 EI α (-2 + L1 α)) Cosh[L2 α] + 4 EI α2 (12 EI α2 + k L12 (3 - 4 L1 α)) Sinh[L2 α])) /
(2 EI α3 (2 k2 L14 - 96 EI2 α4 + (k2 L14 - 24 EI k L12 α2 + 48 EI2 α4) Cos[2 L2 α] + (k2 L14 + 24 EI k L12 α2 + 48 EI2 α4) Cosh[2 L2 α] +
8 EI k L1 α ((-3 + 2 L12 α2) Sin[2 L2 α] + (3 + 2 L12 α2) Sinh[2 L2 α])) +
(e-x2 α Cos[x2 α] (2 EI α2 (F2 (12 (-1 + e2L2 α) EI α2 + k L12 (-3 + 2 (2 + e2L2 α) L1 α)) +
F1 (-k L12 (-3 + (2 + e2L2 α) L1 α) + 12 EI α2 (-1 + e2L2 α (1 + L1 α))) - L1 (F2 k L1 (3 - 2 L1 α) + F1 (12 EI α3 + k L1 (-3 + L1 α))) Cos[2 L2 α] +
6 eL2 α F3 (k L12 - 4 EI α2) Sin[L2 α] + α (- (F1 - 2 F2) k L13 - 12 EI (F1 + F2) α + 12 EI F1 L1 α2) Sin[2 L2 α]) -
F3 Cos[L2 α] (k L12 (k L12 + 4 EI α2 (3 - 4 L1 α)) Cosh[L2 α] - 12 EI α (4 EI α3 + k L1 (-2 + L1 α)) Sinh[L2 α])) /
(2 EI α3 (2 k2 L14 - 96 EI2 α4 + (k2 L14 - 24 EI k L12 α2 + 48 EI2 α4) Cos[2 L2 α] + (k2 L14 + 24 EI k L12 α2 + 48 EI2 α4) Cosh[2 L2 α] +
8 EI k L1 α ((-3 + 2 L12 α2) Sin[2 L2 α] + (3 + 2 L12 α2) Sinh[2 L2 α])) +
(e-2L2 α+x2 α Cos[x2 α] (4 EI α2 (α (- (F1 - 2 F2) k L13 - 12 EI (F1 + F2) α + 12 EI F1 L1 α2) -
e2L2 α L1 (3 (F1 - F2) k L1 + (F1 - 2 F2) k L12 α + 12 EI F1 α3) Cos[2 L2 α] + 6 eL2 α F3 (k L12 - 4 EI α2) Sin[L2 α] + e2L2 α
(-3 (F1 - F2) k L12 - 2 (F1 - 2 F2) k L13 α + 12 EI (F1 + F2) α2 - α (- (F1 - 2 F2) k L13 + 12 EI (F1 + F2) α + 12 EI F1 L1 α2) Sin[2 L2 α])) +
2 e2L2 α F3 Cos[L2 α] (k L12 (k L12 + 4 EI α2 (3 + 4 L1 α)) Cosh[L2 α] + 12 EI α (4 EI α3 + k L1 (2 + L1 α)) Sinh[L2 α])) /
(4 EI α3 (2 k2 L14 - 96 EI2 α4 + (k2 L14 - 24 EI k L12 α2 + 48 EI2 α4) Cos[2 L2 α] + (k2 L14 + 24 EI k L12 α2 + 48 EI2 α4) Cosh[2 L2 α] +
8 EI k L1 α ((-3 + 2 L12 α2) Sin[2 L2 α] + (3 + 2 L12 α2) Sinh[2 L2 α]))

```

## Taglio

Sul tratto 1, il taglio  $V_1$  si determina considerando, nella generica sezione di ascissa  $x_1$ , la risultante delle forze applicate alla sua sinistra (Figura 2). Per il tratto 2, invece, il taglio  $V_2$  si ottiene tramite la derivata terza della funzione  $w_2$ .

```

In[20]:= V1sol = k ((w1sol /. x1 -> 0) x1 + (w1sol - (w1sol /. x1 -> 0)) x1 / 2) - F1
V2sol = -EI D[w2sol, {x2, 3}]

```

$$\begin{aligned}
\text{Out}[20]= & -F1 + \\
& k \left( (3 x1^2 (2 (-F1 + F2) k L1^2 \alpha + \alpha (- (F1 - F2) k L1^2 + 4 EI (F1 + F2) \alpha^2) \text{Cos}[2 L2 \alpha] - \alpha ((F1 - F2) k L1^2 + 4 EI (F1 + F2) \alpha^2) \text{Cosh}[2 L2 \alpha] + 4 F3 k L1 \right. \\
& \quad \left. \text{Cosh}[L2 \alpha] (L1 \alpha \text{Cos}[L2 \alpha] + \text{Sin}[L2 \alpha]) - 8 EI F1 L1 \alpha^4 \text{Sin}[2 L2 \alpha] + 4 F3 (k L1 \text{Cos}[L2 \alpha] + 4 EI \alpha^3 \text{Sin}[L2 \alpha]) \text{Sinh}[L2 \alpha] - \right. \\
& \quad \left. 8 EI F1 L1 \alpha^4 \text{Sinh}[2 L2 \alpha]) \right) / (\alpha (2 k^2 L1^4 - 96 EI^2 \alpha^4 + (k^2 L1^4 - 24 EI k L1^2 \alpha^2 + 48 EI^2 \alpha^4) \text{Cos}[2 L2 \alpha] + \\
& \quad (k^2 L1^4 + 24 EI k L1^2 \alpha^2 + 48 EI^2 \alpha^4) \text{Cosh}[2 L2 \alpha] + 8 EI k L1 \alpha ((-3 + 2 L1^2 \alpha^2) \text{Sin}[2 L2 \alpha] + (3 + 2 L1^2 \alpha^2) \text{Sinh}[2 L2 \alpha]))) - \\
& (2 x1 (2 (-2 F1 + F2) k L1^3 \alpha + L1 \alpha (- (2 F1 - F2) k L1^2 + 12 EI (2 F1 + F2) \alpha^2) \text{Cos}[2 L2 \alpha] + L1 \alpha (- (2 F1 - F2) k L1^2 - 12 EI (2 F1 + F2) \alpha^2) \text{Cosh}[ \\
& \quad 2 L2 \alpha] + 2 F3 \text{Cosh}[L2 \alpha] (2 k L1^3 \alpha \text{Cos}[L2 \alpha] + 3 (k L1^2 + 4 EI \alpha^2) \text{Sin}[L2 \alpha]) + 6 (F3 ((k L1^2 - 4 EI \alpha^2) \text{Cos}[L2 \alpha] + 8 EI L1 \alpha^3 \text{Sin}[L2 \alpha]) \\
& \quad \text{Sinh}[L2 \alpha] + 2 EI \alpha^2 ((F1 + F2 - 2 F1 L1^2 \alpha^2) \text{Sin}[2 L2 \alpha] - (F1 + F2 + 2 F1 L1^2 \alpha^2) \text{Sinh}[2 L2 \alpha]))) / \\
& (\alpha (2 k^2 L1^4 - 96 EI^2 \alpha^4 + (k^2 L1^4 - 24 EI k L1^2 \alpha^2 + 48 EI^2 \alpha^4) \text{Cos}[2 L2 \alpha] + (k^2 L1^4 + 24 EI k L1^2 \alpha^2 + 48 EI^2 \alpha^4) \text{Cosh}[2 L2 \alpha] + \\
& \quad 8 EI k L1 \alpha ((-3 + 2 L1^2 \alpha^2) \text{Sin}[2 L2 \alpha] + (3 + 2 L1^2 \alpha^2) \text{Sinh}[2 L2 \alpha])))
\end{aligned}$$

$$\begin{aligned}
\text{Out}[21]= & -EI ((e^{-2 L2 \alpha + x2 \alpha} \text{Cos}[x2 \alpha] \\
& (4 EI \alpha^2 (-3 e^{2 L2 \alpha} ((F1 - F2) k L1^2 + 4 EI (F1 + F2) \alpha^2 + 8 EI F1 L1 \alpha^3) + L1 (F2 k L1 (3 - 2 L1 \alpha) + F1 (12 EI \alpha^3 + k L1 (-3 + L1 \alpha)))) + \\
& e^{L2 \alpha} (-2 F3 k L1^2 (k L1^2 - 12 EI \alpha^2) \text{Cos}[L2 \alpha] + 4 e^{L2 \alpha} EI \alpha^3 (- (F1 - 2 F2) k L1^3 + 12 EI (F1 + F2) \alpha + 12 EI F1 L1 \alpha^2) \text{Cos}[2 L2 \alpha] + \\
& F3 ((1 + e^{2 L2 \alpha}) k^2 L1^4 + 48 (-1 + e^{2 L2 \alpha}) EI^2 \alpha^4 + 8 EI k L1 \alpha (3 - 2 L1^2 \alpha^2 + e^{2 L2 \alpha} (3 + L1 \alpha (3 + 2 L1 \alpha)))) \text{Sin}[L2 \alpha] - \\
& 4 e^{L2 \alpha} EI L1 \alpha^2 (3 (F1 - F2) k L1 + (F1 - 2 F2) k L1^2 \alpha + 12 EI F1 \alpha^3) \text{Sin}[2 L2 \alpha])) / \\
& (2 EI (2 k^2 L1^4 - 96 EI^2 \alpha^4 + (k^2 L1^4 - 24 EI k L1^2 \alpha^2 + 48 EI^2 \alpha^4) \text{Cos}[2 L2 \alpha] + (k^2 L1^4 + 24 EI k L1^2 \alpha^2 + 48 EI^2 \alpha^4) \text{Cosh}[2 L2 \alpha] + \\
& 8 EI k L1 \alpha ((-3 + 2 L1^2 \alpha^2) \text{Sin}[2 L2 \alpha] + (3 + 2 L1^2 \alpha^2) \text{Sinh}[2 L2 \alpha]))) - \\
& (e^{-2 L2 \alpha + x2 \alpha} (4 EI \alpha^2 (-3 e^{2 L2 \alpha} ((F1 - F2) k L1^2 + 4 EI (F1 + F2) \alpha^2 + 8 EI F1 L1 \alpha^3) + L1 (F2 k L1 (3 - 2 L1 \alpha) + F1 (12 EI \alpha^3 + k L1 (-3 + L1 \alpha)))) + \\
& e^{L2 \alpha} (-2 F3 k L1^2 (k L1^2 - 12 EI \alpha^2) \text{Cos}[L2 \alpha] + 4 e^{L2 \alpha} EI \alpha^3 (- (F1 - 2 F2) k L1^3 + 12 EI (F1 + F2) \alpha + 12 EI F1 L1 \alpha^2) \text{Cos}[2 L2 \alpha] + \\
& F3 ((1 + e^{2 L2 \alpha}) k^2 L1^4 + 48 (-1 + e^{2 L2 \alpha}) EI^2 \alpha^4 + 8 EI k L1 \alpha (3 - 2 L1^2 \alpha^2 + e^{2 L2 \alpha} (3 + L1 \alpha (3 + 2 L1 \alpha)))) \text{Sin}[L2 \alpha] - \\
& 4 e^{L2 \alpha} EI L1 \alpha^2 (3 (F1 - F2) k L1 + (F1 - 2 F2) k L1^2 \alpha + 12 EI F1 \alpha^3) \text{Sin}[2 L2 \alpha])) \text{Sin}[x2 \alpha]) / \\
& (2 EI (2 k^2 L1^4 - 96 EI^2 \alpha^4 + (k^2 L1^4 - 24 EI k L1^2 \alpha^2 + 48 EI^2 \alpha^4) \text{Cos}[2 L2 \alpha] + (k^2 L1^4 + 24 EI k L1^2 \alpha^2 + 48 EI^2 \alpha^4) \text{Cosh}[2 L2 \alpha] + \\
& 8 EI k L1 \alpha ((-3 + 2 L1^2 \alpha^2) \text{Sin}[2 L2 \alpha] + (3 + 2 L1^2 \alpha^2) \text{Sinh}[2 L2 \alpha]))) + \\
& (e^{-x2 \alpha} \text{Cos}[x2 \alpha] (-e^{L2 \alpha} F3 k L1^2 (k L1^2 - 12 EI \alpha^2) \text{Cos}[L2 \alpha] + 2 EI \alpha^2 (-3 (F1 - F2) k L1^2 - 12 EI (F1 + F2) \alpha^2 + 24 EI F1 L1 \alpha^3 - \\
& e^{2 L2 \alpha} L1 (3 (F1 - F2) k L1 + (F1 - 2 F2) k L1^2 \alpha + 12 EI F1 \alpha^3) + \alpha ((F1 - 2 F2) k L1^3 + 12 EI (F1 + F2) \alpha - 12 EI F1 L1 \alpha^2) \text{Cos}[2 L2 \alpha]) - \\
& 2 EI L1 \alpha^2 (F2 k L1 (3 - 2 L1 \alpha) + F1 (12 EI \alpha^3 + k L1 (-3 + L1 \alpha))) \text{Sin}[2 L2 \alpha] + \\
& F3 \text{Sin}[L2 \alpha] (-k L1 (k L1^3 + 12 EI \alpha (-2 + L1 \alpha)) \text{Cosh}[L2 \alpha] + 4 EI \alpha^2 (12 EI \alpha^2 + k L1^2 (3 - 4 L1 \alpha)) \text{Sinh}[L2 \alpha]))) / \\
& (EI (2 k^2 L1^4 - 96 EI^2 \alpha^4 + (k^2 L1^4 - 24 EI k L1^2 \alpha^2 + 48 EI^2 \alpha^4) \text{Cos}[2 L2 \alpha] + (k^2 L1^4 + 24 EI k L1^2 \alpha^2 + 48 EI^2 \alpha^4) \text{Cosh}[2 L2 \alpha] + \\
& 8 EI k L1 \alpha ((-3 + 2 L1^2 \alpha^2) \text{Sin}[2 L2 \alpha] + (3 + 2 L1^2 \alpha^2) \text{Sinh}[2 L2 \alpha]))) + \\
& (e^{-x2 \alpha} \text{Sin}[x2 \alpha] (-e^{L2 \alpha} F3 k L1^2 (k L1^2 - 12 EI \alpha^2) \text{Cos}[L2 \alpha] + 2 EI \alpha^2 (-3 (F1 - F2) k L1^2 - 12 EI (F1 + F2) \alpha^2 + 24 EI F1 L1 \alpha^3 - \\
& e^{2 L2 \alpha} L1 (3 (F1 - F2) k L1 + (F1 - 2 F2) k L1^2 \alpha + 12 EI F1 \alpha^3) + \alpha ((F1 - 2 F2) k L1^3 + 12 EI (F1 + F2) \alpha - 12 EI F1 L1 \alpha^2) \text{Cos}[2 L2 \alpha]) - \\
& 2 EI L1 \alpha^2 (F2 k L1 (3 - 2 L1 \alpha) + F1 (12 EI \alpha^3 + k L1 (-3 + L1 \alpha))) \text{Sin}[2 L2 \alpha] + \\
& F3 \text{Sin}[L2 \alpha] (-k L1 (k L1^3 + 12 EI \alpha (-2 + L1 \alpha)) \text{Cosh}[L2 \alpha] + 4 EI \alpha^2 (12 EI \alpha^2 + k L1^2 (3 - 4 L1 \alpha)) \text{Sinh}[L2 \alpha]))) / \\
& (EI (2 k^2 L1^4 - 96 EI^2 \alpha^4 + (k^2 L1^4 - 24 EI k L1^2 \alpha^2 + 48 EI^2 \alpha^4) \text{Cos}[2 L2 \alpha] + (k^2 L1^4 + 24 EI k L1^2 \alpha^2 + 48 EI^2 \alpha^4) \text{Cosh}[2 L2 \alpha] +
\end{aligned}$$



$$\begin{aligned}
& 8 EI k L1 \alpha ((-3 + 2 L1^2 \alpha^2) \sin[2 L2 \alpha] + (3 + 2 L1^2 \alpha^2) \sinh[2 L2 \alpha])) + \\
& (e^{-x2 \alpha} \cos[x2 \alpha] (2 EI \alpha^2 (F2 (12 (-1 + e^{2 L2 \alpha}) EI \alpha^2 + k L1^2 (-3 + 2 (2 + e^{2 L2 \alpha}) L1 \alpha)) + \\
& \quad F1 (-k L1^2 (-3 + (2 + e^{2 L2 \alpha}) L1 \alpha) + 12 EI \alpha^2 (-1 + e^{2 L2 \alpha} (1 + L1 \alpha))) - L1 (F2 k L1 (3 - 2 L1 \alpha) + F1 (12 EI \alpha^3 + k L1 (-3 + L1 \alpha))) \\
& \quad \cos[2 L2 \alpha] + 6 e^{L2 \alpha} F3 (k L1^2 - 4 EI \alpha^2) \sin[L2 \alpha] + \alpha (- (F1 - 2 F2) k L1^3 - 12 EI (F1 + F2) \alpha + 12 EI F1 L1 \alpha^2) \sin[2 L2 \alpha]) - \\
& \quad F3 \cos[L2 \alpha] (k L1^2 (k L1^2 + 4 EI \alpha^2 (3 - 4 L1 \alpha)) \cosh[L2 \alpha] - 12 EI \alpha (4 EI \alpha^3 + k L1 (-2 + L1 \alpha)) \sinh[L2 \alpha])) / \\
& (EI (2 k^2 L1^4 - 96 EI^2 \alpha^4 + (k^2 L1^4 - 24 EI k L1^2 \alpha^2 + 48 EI^2 \alpha^4) \cos[2 L2 \alpha] + (k^2 L1^4 + 24 EI k L1^2 \alpha^2 + 48 EI^2 \alpha^4) \cosh[2 L2 \alpha] + \\
& \quad 8 EI k L1 \alpha ((-3 + 2 L1^2 \alpha^2) \sin[2 L2 \alpha] + (3 + 2 L1^2 \alpha^2) \sinh[2 L2 \alpha])) - \\
& (e^{-x2 \alpha} \sin[x2 \alpha] (2 EI \alpha^2 (F2 (12 (-1 + e^{2 L2 \alpha}) EI \alpha^2 + k L1^2 (-3 + 2 (2 + e^{2 L2 \alpha}) L1 \alpha)) + \\
& \quad F1 (-k L1^2 (-3 + (2 + e^{2 L2 \alpha}) L1 \alpha) + 12 EI \alpha^2 (-1 + e^{2 L2 \alpha} (1 + L1 \alpha))) - L1 (F2 k L1 (3 - 2 L1 \alpha) + F1 (12 EI \alpha^3 + k L1 (-3 + L1 \alpha))) \\
& \quad \cos[2 L2 \alpha] + 6 e^{L2 \alpha} F3 (k L1^2 - 4 EI \alpha^2) \sin[L2 \alpha] + \alpha (- (F1 - 2 F2) k L1^3 - 12 EI (F1 + F2) \alpha + 12 EI F1 L1 \alpha^2) \sin[2 L2 \alpha]) - \\
& \quad F3 \cos[L2 \alpha] (k L1^2 (k L1^2 + 4 EI \alpha^2 (3 - 4 L1 \alpha)) \cosh[L2 \alpha] - 12 EI \alpha (4 EI \alpha^3 + k L1 (-2 + L1 \alpha)) \sinh[L2 \alpha])) / \\
& (EI (2 k^2 L1^4 - 96 EI^2 \alpha^4 + (k^2 L1^4 - 24 EI k L1^2 \alpha^2 + 48 EI^2 \alpha^4) \cos[2 L2 \alpha] + (k^2 L1^4 + 24 EI k L1^2 \alpha^2 + 48 EI^2 \alpha^4) \cosh[2 L2 \alpha] + \\
& \quad 8 EI k L1 \alpha ((-3 + 2 L1^2 \alpha^2) \sin[2 L2 \alpha] + (3 + 2 L1^2 \alpha^2) \sinh[2 L2 \alpha])) - \\
& (e^{-2 L2 \alpha + x2 \alpha} \cos[x2 \alpha] (4 EI \alpha^2 (\alpha (- (F1 - 2 F2) k L1^3 - 12 EI (F1 + F2) \alpha + 12 EI F1 L1 \alpha^2) - \\
& \quad e^{2 L2 \alpha} L1 (3 (F1 - F2) k L1 + (F1 - 2 F2) k L1^2 \alpha + 12 EI F1 \alpha^3) \cos[2 L2 \alpha] + 6 e^{L2 \alpha} F3 (k L1^2 - 4 EI \alpha^2) \sin[L2 \alpha] + e^{2 L2 \alpha} \\
& \quad (-3 (F1 - F2) k L1^2 - 2 (F1 - 2 F2) k L1^3 \alpha + 12 EI (F1 + F2) \alpha^2 - \alpha (- (F1 - 2 F2) k L1^3 + 12 EI (F1 + F2) \alpha + 12 EI F1 L1 \alpha^2) \sin[2 L2 \alpha])) + \\
& \quad 2 e^{2 L2 \alpha} F3 \cos[L2 \alpha] (k L1^2 (k L1^2 + 4 EI \alpha^2 (3 + 4 L1 \alpha)) \cosh[L2 \alpha] + 12 EI \alpha (4 EI \alpha^3 + k L1 (2 + L1 \alpha)) \sinh[L2 \alpha])) / \\
& (2 EI (2 k^2 L1^4 - 96 EI^2 \alpha^4 + (k^2 L1^4 - 24 EI k L1^2 \alpha^2 + 48 EI^2 \alpha^4) \cos[2 L2 \alpha] + (k^2 L1^4 + 24 EI k L1^2 \alpha^2 + 48 EI^2 \alpha^4) \cosh[2 L2 \alpha] + \\
& \quad 8 EI k L1 \alpha ((-3 + 2 L1^2 \alpha^2) \sin[2 L2 \alpha] + (3 + 2 L1^2 \alpha^2) \sinh[2 L2 \alpha])) - \\
& (e^{-2 L2 \alpha + x2 \alpha} \sin[x2 \alpha] (4 EI \alpha^2 (\alpha (- (F1 - 2 F2) k L1^3 - 12 EI (F1 + F2) \alpha + 12 EI F1 L1 \alpha^2) - \\
& \quad e^{2 L2 \alpha} L1 (3 (F1 - F2) k L1 + (F1 - 2 F2) k L1^2 \alpha + 12 EI F1 \alpha^3) \cos[2 L2 \alpha] + 6 e^{L2 \alpha} F3 (k L1^2 - 4 EI \alpha^2) \sin[L2 \alpha] + e^{2 L2 \alpha} \\
& \quad (-3 (F1 - F2) k L1^2 - 2 (F1 - 2 F2) k L1^3 \alpha + 12 EI (F1 + F2) \alpha^2 - \alpha (- (F1 - 2 F2) k L1^3 + 12 EI (F1 + F2) \alpha + 12 EI F1 L1 \alpha^2) \sin[2 L2 \alpha])) + \\
& \quad 2 e^{2 L2 \alpha} F3 \cos[L2 \alpha] (k L1^2 (k L1^2 + 4 EI \alpha^2 (3 + 4 L1 \alpha)) \cosh[L2 \alpha] + 12 EI \alpha (4 EI \alpha^3 + k L1 (2 + L1 \alpha)) \sinh[L2 \alpha])) / \\
& (2 EI (2 k^2 L1^4 - 96 EI^2 \alpha^4 + (k^2 L1^4 - 24 EI k L1^2 \alpha^2 + 48 EI^2 \alpha^4) \cos[2 L2 \alpha] + (k^2 L1^4 + 24 EI k L1^2 \alpha^2 + 48 EI^2 \alpha^4) \cosh[2 L2 \alpha] + \\
& \quad 8 EI k L1 \alpha ((-3 + 2 L1^2 \alpha^2) \sin[2 L2 \alpha] + (3 + 2 L1^2 \alpha^2) \sinh[2 L2 \alpha]))))
\end{aligned}$$

## Momento flettente

Similmente, sul tratto 1, il momento flettente  $M_1$  si determina considerando, nella generica sezione di ascissa  $x_1$ , il momento risultante delle forze applicate alla sua sinistra (Figura 2). Per il tratto 2, invece, il momento flettente  $M_2$  si ottiene tramite la derivata seconda della funzione  $w_2$ .

```
In[22]:= M1sol = k ((w1sol /. x1 -> 0) x1^2 / 2 + (w1sol - (w1sol /. x1 -> 0)) x1^2 / 6) - F1 x1
M2sol = -EI D[w2sol, {x2, 2}]
```

General::spell1 : Possible spelling error: new symbol name "M1sol" is similar to existing symbol "V1sol". More...

```
Out[22]= -F1 x1 +
k ((x1^3 (2 (-F1 + F2) k L1^2 α + α (- (F1 - F2) k L1^2 + 4 EI (F1 + F2) α^2) Cos[2 L2 α] - α ((F1 - F2) k L1^2 + 4 EI (F1 + F2) α^2) Cosh[2 L2 α] + 4 F3 k L1
Cosh[L2 α] (L1 α Cos[L2 α] + Sin[L2 α]) - 8 EI F1 L1 α^4 Sin[2 L2 α] + 4 F3 (k L1 Cos[L2 α] + 4 EI α^3 Sin[L2 α]) Sinh[L2 α] -
8 EI F1 L1 α^4 Sinh[2 L2 α])) / (α (2 k^2 L1^4 - 96 EI^2 α^4 + (k^2 L1^4 - 24 EI k L1^2 α^2 + 48 EI^2 α^4) Cos[2 L2 α] +
(k^2 L1^4 + 24 EI k L1^2 α^2 + 48 EI^2 α^4) Cosh[2 L2 α] + 8 EI k L1 α ((-3 + 2 L1^2 α^2) Sin[2 L2 α] + (3 + 2 L1^2 α^2) Sinh[2 L2 α]))) -
(x1^2 (2 (-2 F1 + F2) k L1^3 α + L1 α (- (2 F1 - F2) k L1^2 + 12 EI (2 F1 + F2) α^2) Cos[2 L2 α] + L1 α (- (2 F1 - F2) k L1^2 - 12 EI (2 F1 + F2) α^2) Cosh[
2 L2 α] + 2 F3 Cosh[L2 α] (2 k L1^3 α Cos[L2 α] + 3 (k L1^2 + 4 EI α^2) Sin[L2 α]) + 6 (F3 ((k L1^2 - 4 EI α^2) Cos[L2 α] + 8 EI L1 α^3 Sin[L2 α])
Sinh[L2 α] + 2 EI α^2 ((F1 + F2 - 2 F1 L1^2 α^2) Sin[2 L2 α] - (F1 + F2 + 2 F1 L1^2 α^2) Sinh[2 L2 α]))) /
(α (2 k^2 L1^4 - 96 EI^2 α^4 + (k^2 L1^4 - 24 EI k L1^2 α^2 + 48 EI^2 α^4) Cos[2 L2 α] + (k^2 L1^4 + 24 EI k L1^2 α^2 + 48 EI^2 α^4) Cosh[2 L2 α] +
8 EI k L1 α ((-3 + 2 L1^2 α^2) Sin[2 L2 α] + (3 + 2 L1^2 α^2) Sinh[2 L2 α])))
```

General::spell1 : Possible spelling error: new symbol name "M2sol" is similar to existing symbol "V2sol". More...

$$\begin{aligned}
\text{Out}[23]= & -EI \left( (e^{-2L_2\alpha+x_2\alpha} \cos[x_2\alpha] \right. \\
& (4EI\alpha^2(-3e^{2L_2\alpha}(F_1-F_2)kL_1^2+4EI(F_1+F_2)\alpha^2+8EIF_1L_1\alpha^3)+L_1(F_2kL_1(3-2L_1\alpha)+F_1(12EI\alpha^3+kL_1(-3+L_1\alpha))))+ \\
& e^{L_2\alpha}(-2F_3kL_1^2(kL_1^2-12EI\alpha^2)\cos[L_2\alpha]+4e^{L_2\alpha}EI\alpha^3(-(F_1-2F_2)kL_1^3+12EI(F_1+F_2)\alpha+12EIF_1L_1\alpha^2)\cos[2L_2\alpha]+ \\
& F_3((1+e^{2L_2\alpha})k^2L_1^4+48(-1+e^{2L_2\alpha})EI^2\alpha^4+8EI kL_1\alpha(3-2L_1^2\alpha^2+e^{2L_2\alpha}(3+L_1\alpha(3+2L_1\alpha))))\sin[L_2\alpha]- \\
& 4e^{L_2\alpha}EI L_1\alpha^2(3(F_1-F_2)kL_1+(F_1-2F_2)kL_1^2\alpha+12EIF_1\alpha^3)\sin[2L_2\alpha])) / \\
& (2EI\alpha(2k^2L_1^4-96EI^2\alpha^4+(k^2L_1^4-24EI kL_1^2\alpha^2+48EI^2\alpha^4)\cos[2L_2\alpha]+(k^2L_1^4+24EI kL_1^2\alpha^2+48EI^2\alpha^4)\cosh[2L_2\alpha]+ \\
& 8EI kL_1\alpha((-3+2L_1^2\alpha^2)\sin[2L_2\alpha]+(3+2L_1^2\alpha^2)\sinh[2L_2\alpha])) - \\
& (e^{-x_2\alpha}\cos[x_2\alpha](-e^{L_2\alpha}F_3kL_1^2(kL_1^2-12EI\alpha^2)\cos[L_2\alpha]+2EI\alpha^2(-3(F_1-F_2)kL_1^2-12EI(F_1+F_2)\alpha^2+24EIF_1L_1\alpha^3- \\
& e^{2L_2\alpha}L_1(3(F_1-F_2)kL_1+(F_1-2F_2)kL_1^2\alpha+12EIF_1\alpha^3))+\alpha((F_1-2F_2)kL_1^3+12EI(F_1+F_2)\alpha-12EIF_1L_1\alpha^2)\cos[2L_2\alpha]) - \\
& 2EI L_1\alpha^2(F_2kL_1(3-2L_1\alpha)+F_1(12EI\alpha^3+kL_1(-3+L_1\alpha)))\sin[2L_2\alpha]+ \\
& F_3\sin[L_2\alpha](-kL_1(kL_1^3+12EI\alpha(-2+L_1\alpha))\cosh[L_2\alpha]+4EI\alpha^2(12EI\alpha^2+kL_1^2(3-4L_1\alpha))\sinh[L_2\alpha])) / \\
& (EI\alpha(2k^2L_1^4-96EI^2\alpha^4+(k^2L_1^4-24EI kL_1^2\alpha^2+48EI^2\alpha^4)\cos[2L_2\alpha]+(k^2L_1^4+24EI kL_1^2\alpha^2+48EI^2\alpha^4)\cosh[2L_2\alpha]+ \\
& 8EI kL_1\alpha((-3+2L_1^2\alpha^2)\sin[2L_2\alpha]+(3+2L_1^2\alpha^2)\sinh[2L_2\alpha])) + \\
& (e^{-x_2\alpha}\sin[x_2\alpha](2EI\alpha^2(F_2(12(-1+e^{2L_2\alpha})EI\alpha^2+kL_1^2(-3+2(2+e^{2L_2\alpha})L_1\alpha))+ \\
& F_1(-kL_1^2(-3+(2+e^{2L_2\alpha})L_1\alpha)+12EI\alpha^2(-1+e^{2L_2\alpha}(1+L_1\alpha)))-L_1(F_2kL_1(3-2L_1\alpha)+F_1(12EI\alpha^3+kL_1(-3+L_1\alpha))) \\
& \cos[2L_2\alpha]+6e^{L_2\alpha}F_3(kL_1^2-4EI\alpha^2)\sin[L_2\alpha]+\alpha(-(F_1-2F_2)kL_1^3-12EI(F_1+F_2)\alpha+12EIF_1L_1\alpha^2)\sin[2L_2\alpha]) - \\
& F_3\cos[L_2\alpha](kL_1^2(kL_1^2+4EI\alpha^2(3-4L_1\alpha))\cosh[L_2\alpha]-12EI\alpha(4EI\alpha^3+kL_1(-2+L_1\alpha))\sinh[L_2\alpha])) / \\
& (EI\alpha(2k^2L_1^4-96EI^2\alpha^4+(k^2L_1^4-24EI kL_1^2\alpha^2+48EI^2\alpha^4)\cos[2L_2\alpha]+(k^2L_1^4+24EI kL_1^2\alpha^2+48EI^2\alpha^4)\cosh[2L_2\alpha]+ \\
& 8EI kL_1\alpha((-3+2L_1^2\alpha^2)\sin[2L_2\alpha]+(3+2L_1^2\alpha^2)\sinh[2L_2\alpha])) - \\
& (e^{-2L_2\alpha+x_2\alpha}\sin[x_2\alpha](4EI\alpha^2(\alpha(-(F_1-2F_2)kL_1^3-12EI(F_1+F_2)\alpha+12EIF_1L_1\alpha^2)- \\
& e^{2L_2\alpha}L_1(3(F_1-F_2)kL_1+(F_1-2F_2)kL_1^2\alpha+12EIF_1\alpha^3)\cos[2L_2\alpha]+6e^{L_2\alpha}F_3(kL_1^2-4EI\alpha^2)\sin[L_2\alpha]+e^{2L_2\alpha} \\
& (-3(F_1-F_2)kL_1^2-2(F_1-2F_2)kL_1^3\alpha+12EI(F_1+F_2)\alpha^2-\alpha(-(F_1-2F_2)kL_1^3+12EI(F_1+F_2)\alpha+12EIF_1L_1\alpha^2)\sin[2L_2\alpha]))+ \\
& 2e^{2L_2\alpha}F_3\cos[L_2\alpha](kL_1^2(kL_1^2+4EI\alpha^2(3+4L_1\alpha))\cosh[L_2\alpha]+12EI\alpha(4EI\alpha^3+kL_1(2+L_1\alpha))\sinh[L_2\alpha])) / \\
& (2EI\alpha(2k^2L_1^4-96EI^2\alpha^4+(k^2L_1^4-24EI kL_1^2\alpha^2+48EI^2\alpha^4)\cos[2L_2\alpha]+(k^2L_1^4+24EI kL_1^2\alpha^2+48EI^2\alpha^4)\cosh[2L_2\alpha]+ \\
& 8EI kL_1\alpha((-3+2L_1^2\alpha^2)\sin[2L_2\alpha]+(3+2L_1^2\alpha^2)\sinh[2L_2\alpha]))))
\end{aligned}$$

## Esempi

### ■ 1° Caso: $L_2 = \lambda/8$

```
In[24]:= L1num = 4000;
         L2num = 6000;
         F1num = 500000;
         F2num = 1000000;
         F3num = 500000;
         k0num = 0.01;
         bnum = 1000;
         hnum = 1504.58;
         Ecnun = 30000;
         EInum = Ecnun bnum hnum ^ 3 / 12;
         knum = k0num bnum;
         anum = (knum / EInum / 4) ^ .25
         lambdanum = 2 Pi / anum
```

```
Out[35]= 0.0001309
```

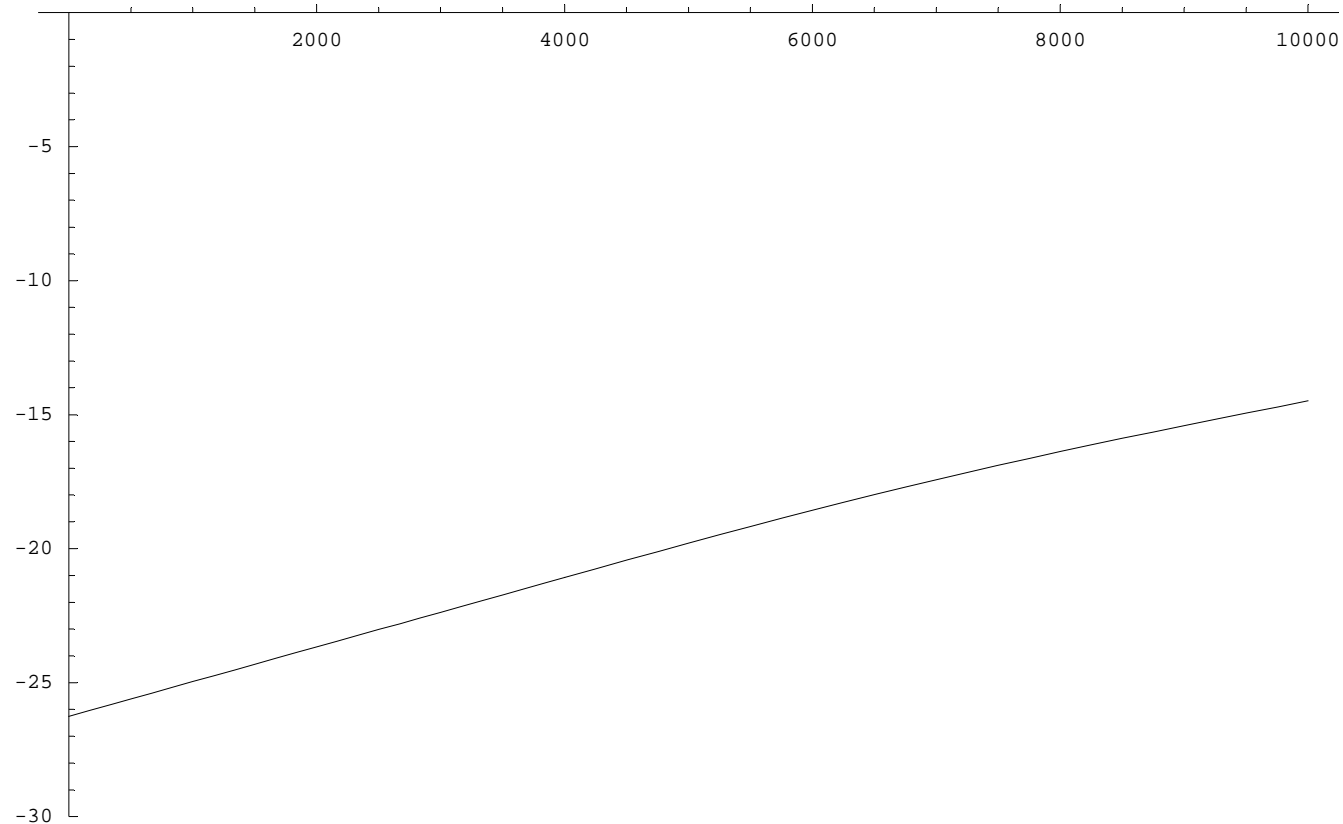
```
Out[36]= 48000.
```

```
In[37]:= w1solNum = w1sol /. {L1 -> L1num, L2 -> L2num, F1 -> F1num, F2 -> F2num, F3 -> F3num, EI -> EInum, k -> knum, alpha -> anum}
         w2solNum = w2sol /. {L1 -> L1num, L2 -> L2num, F1 -> F1num, F2 -> F2num, F3 -> F3num, EI -> EInum, k -> knum, alpha -> anum}
```

```
Out[37]= 26.2563 - 0.00129444 x1
```

```
Out[38]= 21.3858 e-1.5708+0.0001309 x2 Cos[0.0001309 x2] + 16.6328 e-0.0001309 x2 Cos[0.0001309 x2] +
         5.83777 e-1.5708+0.0001309 x2 Sin[0.0001309 x2] + 1.08479 e-0.0001309 x2 Sin[0.0001309 x2]
```

```
In[39]:= Show[Plot[-w1solNum, {x1, 0, L1num}, PlotRange -> {-30, 0}],  
Plot[-w2solNum /. x2 -> x - L1num, {x, L1num, L2num + L1num}, PlotRange -> {-30, 0}]]
```



```
Out[39]= - Graphics -
```

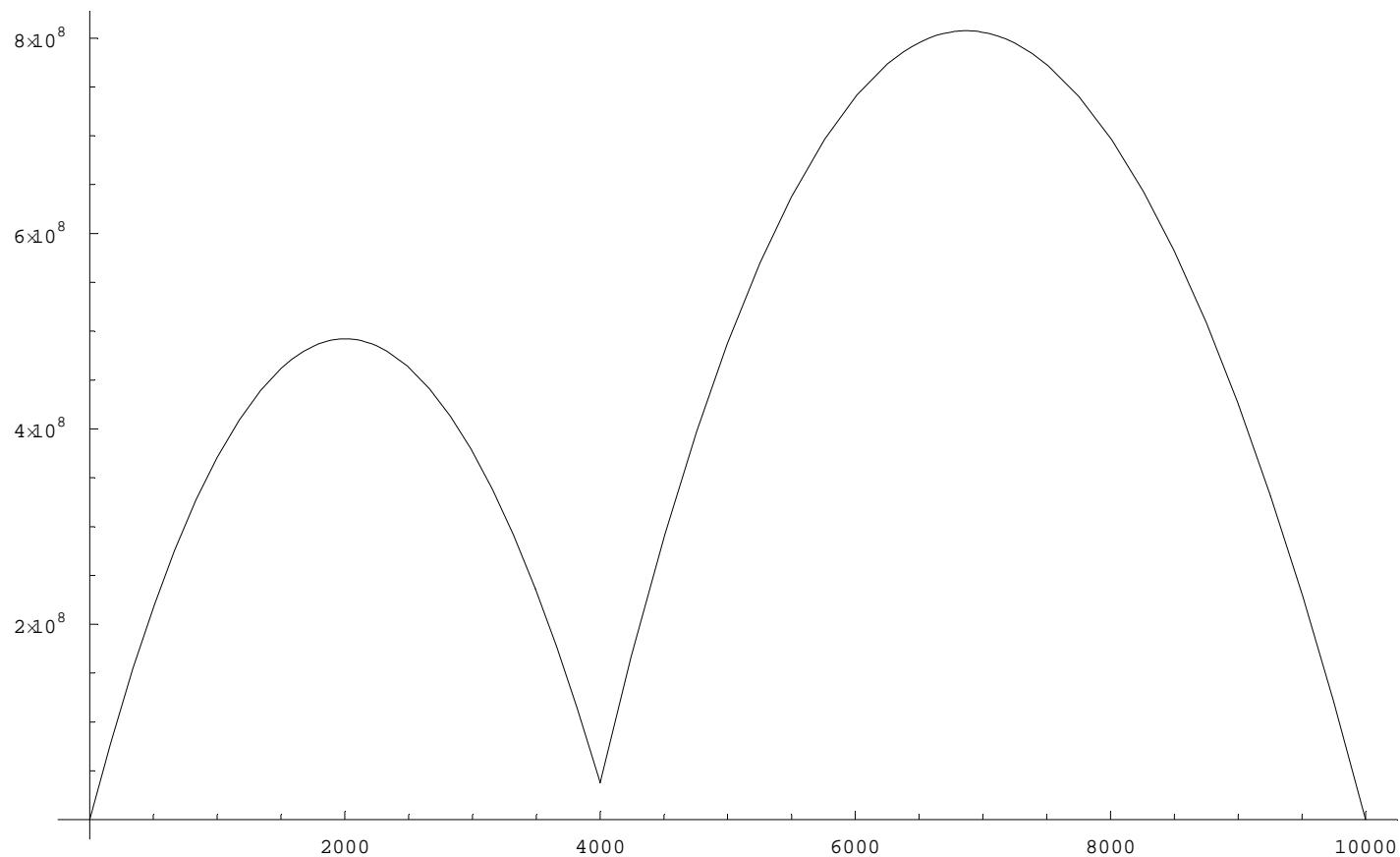
Figura 3: linea elastica del sistema di travi.

```
In[40]:= M1solNum = M1sol /. {L1 -> L1num, L2 -> L2num, F1 -> F1num, F2 -> F2num, F3 -> F3num, EI -> EInum, k -> knum,  $\alpha$  -> anum}  
M2solNum = M2sol /. {L1 -> L1num, L2 -> L2num, F1 -> F1num, F2 -> F2num, F3 -> F3num, EI -> EInum, k -> knum,  $\alpha$  -> anum}
```

```
Out[40]= -500000 x1 + 10. (13.1281 x12 - 0.00021574 x13)
```

```
Out[41]= -8.51502  $\times 10^{15}$  (2.00057  $\times 10^{-7}$  e-1.5708+0.0001309 x2 Cos[0.0001309 x2] - 3.71752  $\times 10^{-8}$  e-0.0001309 x2 Cos[0.0001309 x2] -  
7.3288  $\times 10^{-7}$  e-1.5708+0.0001309 x2 Sin[0.0001309 x2] + 5.69997  $\times 10^{-7}$  e-0.0001309 x2 Sin[0.0001309 x2])
```

```
In[42]:= Show[Plot[-M1solNum, {x1, 0, L1num}], Plot[-M2solNum /. x2 -> x - L1num, {x, L1num, L2num + L1num}]]
```



```
Out[42]= - Graphics -
```

Figura 4: linea elastica del sistema di travi.

Osservazioni: il caso di  $L_2 = \lambda/8$  determina un comportamento sostanzialmente assimilabile a quello di un'unica trave rigida: si osserva, infatti, che la linea elastica è sostanzialmente rappresentata da un segmento di retta (Figura 3).

## ■ 2° Caso: $L = \lambda/2$

```
In[79]:= L1num = 4000;
          L2num = 24000;
          F1num = 500000;
          F2num = 1000000;
          F3num = 500000;
          k0num = 0.01;
          bnum = 1000;
          hnum = 1504.58;
          Ecnun = 30000;
          EInum = Ecnun bnum hnum ^ 3 / 12;
          knum = k0num bnum;
          anum = (knum / EInum / 4) ^ .25
          lambdanum = 2 Pi / anum
```

```
Out[90]= 0.0001309
```

```
Out[91]= 48000.
```

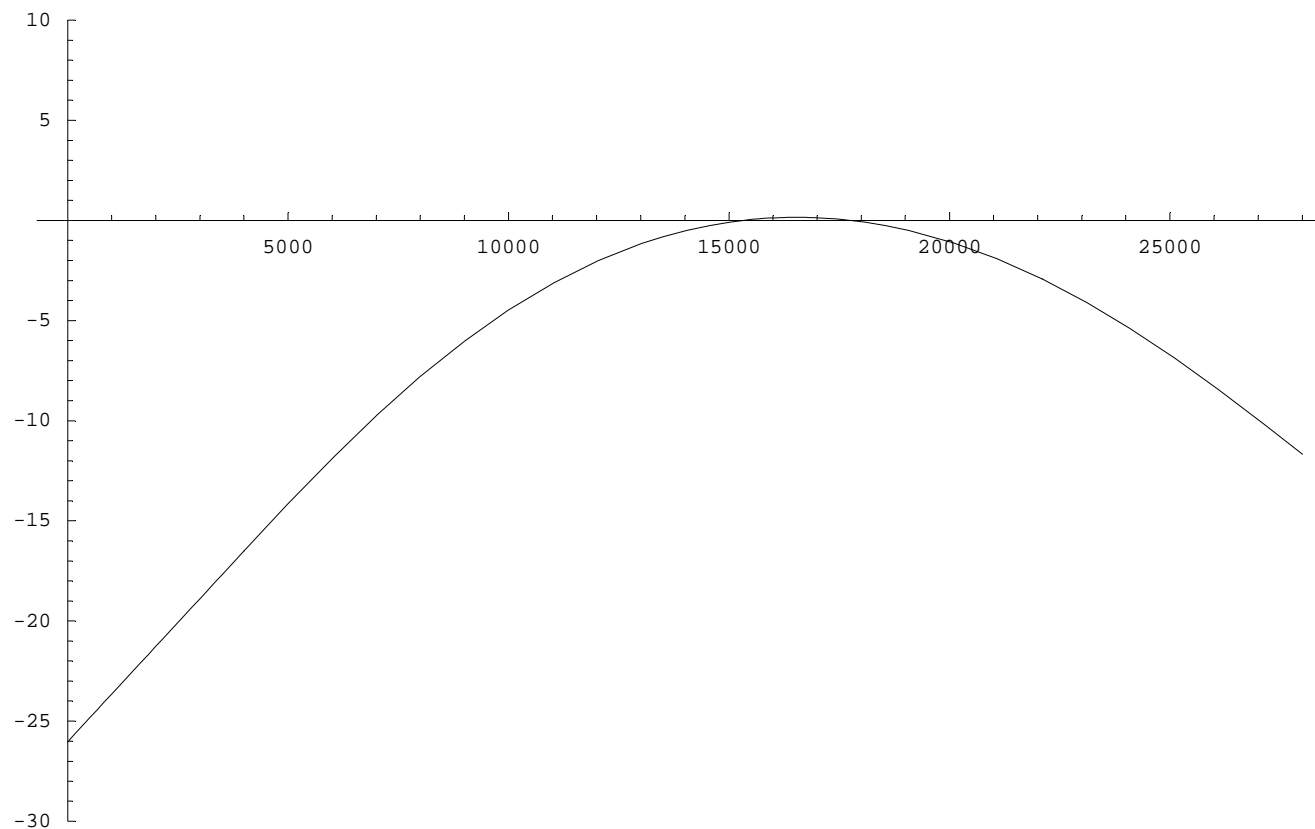
```
In[92]:= w1solNum = w1sol /. {L1 -> L1num, L2 -> L2num, F1 -> F1num, F2 -> F2num, F3 -> F3num, EI -> EInum, k -> knum, alpha -> anum}
          w2solNum = w2sol /. {L1 -> L1num, L2 -> L2num, F1 -> F1num, F2 -> F2num, F3 -> F3num, EI -> EInum, k -> knum, alpha -> anum}
```

```
Out[92]= 26.029 - 0.00237905 x1
```

```
Out[93]= -287.039 e-6.28318+0.0001309 x2 Cos[0.0001309 x2] + 17.0488 e-0.0001309 x2 Cos[0.0001309 x2] -
          0.588433 e-6.28318+0.0001309 x2 Sin[0.0001309 x2] - 0.588641 e-0.0001309 x2 Sin[0.0001309 x2]
```



```
In[94]:= Show[Plot[-w1solNum, {x1, 0, L1num}, PlotRange -> {-30, 10}],  
Plot[-w2solNum /. x2 -> x - L1num, {x, L1num, L2num + L1num}, PlotRange -> {-30, 10}]]
```



```
Out[94]= - Graphics -
```

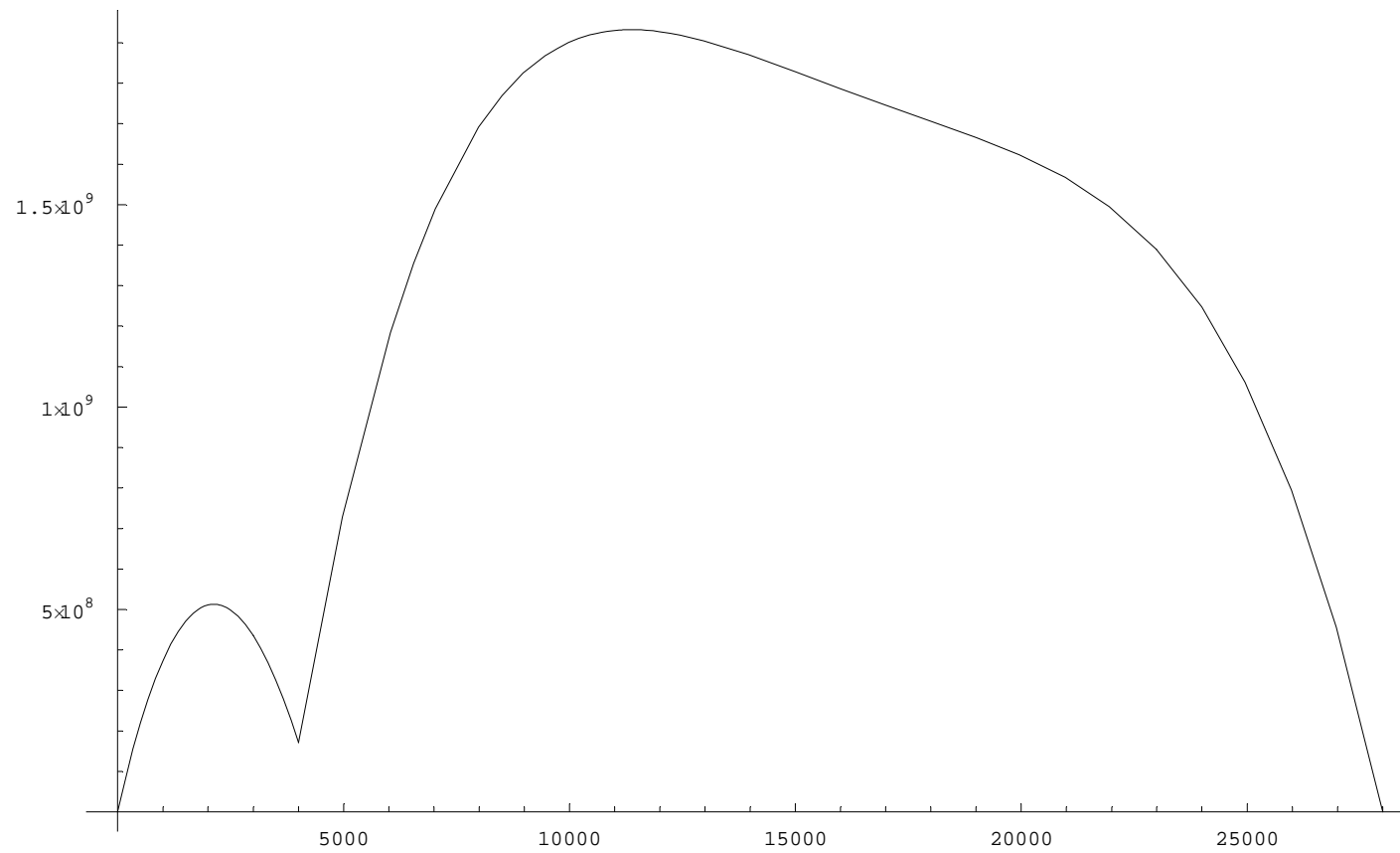
Osservazioni: il caso di  $L_2 = \lambda/4$  evidenzia la maggiore flessibilità del tratto 2, la cui deformata è evidentemente "non lineare" (Figura 5).

```
In[60]:= M1solNum = M1sol /. {L1 -> L1num, L2 -> L2num, F1 -> F1num, F2 -> F2num, F3 -> F3num, EI -> EInum, k -> knum,  $\alpha$  -> anum}  
M2solNum = M2sol /. {L1 -> L1num, L2 -> L2num, F1 -> F1num, F2 -> F2num, F3 -> F3num, EI -> EInum, k -> knum,  $\alpha$  -> anum}
```

```
Out[60]= -500000 x1 + 10. (13.0145 x12 - 0.000396508 x13)
```

```
Out[61]=  $-8.51502 \times 10^{15} (-2.01653 \times 10^{-8} e^{-6.28318+0.0001309 x2} \text{Cos}[0.0001309 x2] + 2.01724 \times 10^{-8} e^{-0.0001309 x2} \text{Cos}[0.0001309 x2] +$   
 $9.83667 \times 10^{-6} e^{-6.28318+0.0001309 x2} \text{Sin}[0.0001309 x2] + 5.84253 \times 10^{-7} e^{-0.0001309 x2} \text{Sin}[0.0001309 x2])$ 
```

```
In[95]:= Show[Plot[-M1solNum, {x1, 0, L1num}], Plot[-M2solNum /. x2 -> x - L1num, {x, L1num, L2num + L1num}]]
```



```
Out[95]= - Graphics -
```