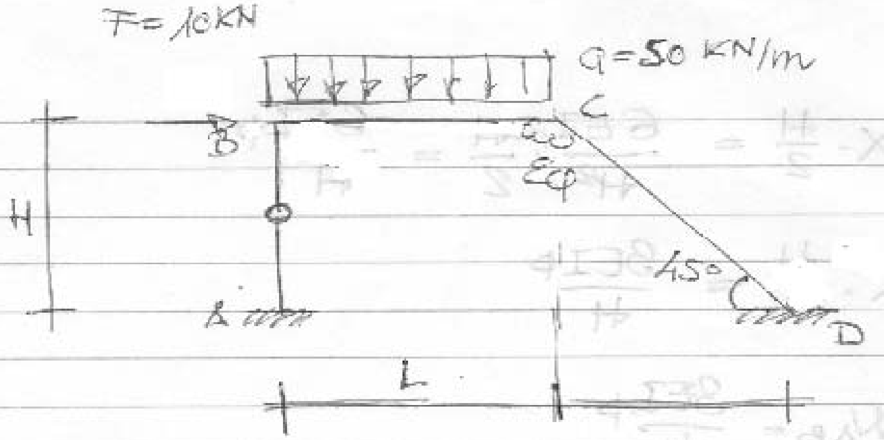
Analisi delle sollecitazioni su **Strutture intelaiate piane**

Esercizio sul tema del **Portale ad un piano ed una campata**



Dati.

$R_{ck} = 25\,MPa$ $E_c = 9500\,(20+8)^{1/3} = 28848\,MPa$

Dati geometrici

$L = 5.00\,m$ $H = 3.00\,m$

Sezioni.

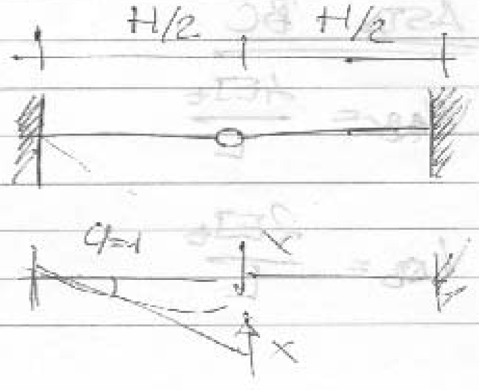
Pilastri $b_p = 30$ $h_p = 50$

Travi $b_t = 30$ $h_t = 60$

$$\varepsilon\varphi = \frac{H}{10\,EI_t}$$

1. Calcolo delle rigidezze degli elementi.

– Asta AB



$$1 \cdot \frac{H}{2} - \frac{XH^3}{24EI_p} = \frac{XH^3}{24EI_p}$$

$$X = \frac{6EI_p}{H^2}$$

$$W_{AB} = X \cdot \frac{H}{2} = \frac{6EI_p}{H^2} \cdot \frac{H}{2} = \frac{3EI_p}{H}$$

$$V_{AB} = X \cdot \frac{H}{2} = \frac{3EI_p}{H}$$

$$W_{BA} = W_{AB} = \frac{3EI_p}{H}$$

$$V_{BA} = \frac{3EI_p}{H}$$

calcolo del valore numerico

$$W_{AB} = \frac{3EI_p}{H} = \frac{3 \cdot 28848 \cdot 3{,}125 \cdot 10^9}{3000} = 9{,}015 \cdot 10^{10}\ \text{Nmm}$$

$$I_p = \frac{b h_p^3}{12} = \frac{300 \cdot 500^3}{12} = 3{,}125 \cdot 10^9\ \text{mm}$$

$$U_{AB} = U_{BA} = \frac{W_{AB} + V_{AB}}{H} = \frac{2 \cdot 9{,}015 \cdot 10^{10}}{3000} = 6{,}01 \cdot 10^7\ \text{N}$$

ASTA BC

$$W_{BC} = \frac{4EI_t}{L} \qquad V_{BC} = \frac{2EI_t}{L}$$

$$V_{CB} = \frac{2EI_t}{L} \qquad W_{CB} = \frac{4EI_t}{L}$$

$$U_{BC} = U_{CB} = \frac{W_{CB} + V_{CB}}{L} - \quad M_{BC} = -\frac{ql^2}{12}$$

$$M_{CB} = \frac{ql^2}{12}$$

- valori numerici

$$W_{BC} = W_{CB} = \frac{4EI_p}{L} = \frac{4 \cdot 28848 \cdot 5.4 \cdot 10^9}{5000} = 1.246 \cdot 10^{11}$$

Nmm

$$I_t = \frac{B_t \cdot h_t^3}{12} = \frac{300 \cdot 600^3}{12} = 5.4 \cdot 10^9$$

$$V_{CB} = V_{BC} = 6.23 \cdot 10^{10} \text{ Nmm}$$

$$U_{CB} = U_{BC} = 3.739 \cdot 10^7 \text{ N}$$

$$M_{BC} = -M_{CB} = -\frac{50 \cdot 5000^2}{12} = -1.047 \cdot 10^8$$

<u>ASTA CD</u>

$$\alpha'_{CD} = \alpha_{CD} + \varepsilon q = \frac{L_{CD}}{8EI_p} + \frac{H}{10EI_t} =$$

$$\frac{3000 \sqrt{2}}{8 \cdot 28848 \cdot 3.125 \cdot 10^9} + \frac{5000}{10 \cdot 28848 \cdot 5.4 \cdot 10^9} =$$

$$1.5687 \cdot 10^{-11} + 3.2097 \cdot 10^{-12} = 1.8897 \cdot 10^{-11}$$

$$\alpha_{DC} = \frac{L_{CD}}{3EJ_D} = 1.5687 \cdot 10^{-11}$$

$$\beta = 7.8135 \cdot 10^{-12}$$

$$W_{CD} = \frac{\alpha_{DC}}{\alpha_{CD} \cdot \alpha_{DC} - \beta^2} = \frac{1.5687 \cdot 10^{-11}}{1.5687 \cdot 10^{-11} \cdot 1.8897 \cdot 10^{-11} - (7.8135 \cdot 10^{-12})^2}$$

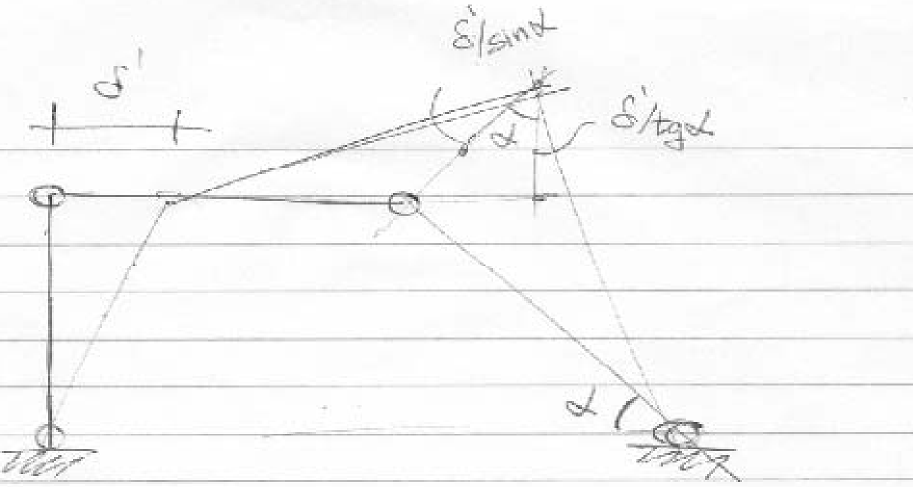
$$= \frac{1.5687 \cdot 10^{-11}}{2.3492 \cdot 10^{-22}} = 6.678 \cdot 10^{10} \ Nmm$$

$$W_{DC} = \frac{\alpha_{CD}}{\alpha_{CD}\alpha_{DC} - \beta^2} = \frac{1.8897 \cdot 10^{-11}}{2.3492 \cdot 10^{-22}} = 8.044 \cdot 10^{10} \ Nmm$$

$$V_{DC} = V_{CD} = \frac{\beta}{\alpha_{CD}\alpha_{DC} - \beta^2} = 3.339 \cdot 10^{10} \ Nmm$$

$$U_{CD} = \frac{W_{CD} + V_{CD}}{\sqrt{2}H} = \frac{6.678 \cdot 10^{10} + 3.339 \cdot 10^{10}}{\sqrt{2} \cdot 3000} = 2.361 \cdot 10^{7}$$

$$U_{DC} = \frac{W_{DC} + V_{DC}}{\sqrt{2}H} = \frac{8.044 \cdot 10^{10} + 3.339 \cdot 10^{10}}{\sqrt{2} \cdot 3000} = 2.683 \cdot 10^{7}$$



Spostamenti virtuali	Spostamenti effettivi
$\delta'_{AB} = \delta'$	$\delta_{AB} = \delta$
$\delta'_{BC} = -\delta'$	$\delta_{BC} = -\delta$
$\delta'_{CD} = \sqrt{2}\,\delta'$	$\delta_{CD} = \sqrt{2}\,\delta$

Spostamenti relativi di asta ($\delta_{AB}, \delta_{BC}, \delta_{CD}; \delta'_{AB}, \delta'_{BC}, \delta'_{CD}$)

	Spostamento nodale (δ, δ')
δ_{AB} (δ'_{AB})	1
δ_{BC} (δ'_{BC})	-1
δ_{CD} (δ'_{CD})	$\sqrt{2}$

SCRITTURA DELLE EQUAZIONI DI EQUILIBRIO

1) $M_{BA} + M_{BC} = 0$

$W_{BA} \cdot \varphi_B - U_{BA} \cdot \delta_{BA} + W_{BC} \varphi_B + V_{BC} \cdot \varphi_C - U_{BC} \cdot \delta_{BC} + \mu_{BC} = 0$

2) $M_{CB} + M_{CD} = 0$

$W_{CB} \cdot \varphi_C + V_{CB} \cdot \varphi_B - U_{CB} \cdot \delta_{CB} + \mu_{CB} + W_{CD} \cdot \varphi_C - U_{CD} \delta_D = 0$

3) $(M_{AB} + M_{BA}) \cdot \frac{\delta'_{BA}}{H} + (M_{BC} + M_{CB}) \frac{\delta'_{BC}}{L} + (M_{CD} + M_{DC}) \cdot \frac{\delta'_{CD}}{\sqrt{2} \cdot H} + F \cdot \delta' - qL \cdot \frac{\delta'_{BC}}{2} = 0$

Scrittura delle equazioni in forma matriciale

$(W_{BA} + W_{BC}) \varphi_B + V_{BC} \varphi_C + (U_{BC} - U_{BA}) \delta = -\mu_{BC}$

$(V_{BC}) \varphi_B + (W_{CB} + W_{CD}) \varphi_B + (U_{CB} - \sqrt{2} U_{CD}) \delta = -\mu_{CB}$

$(M_{AB} + M_{BA}) \frac{\delta'}{H} - (M_{BC} + M_{CB}) \frac{\delta'}{L} + (M_{CD} + M_{DC}) \frac{\sqrt{2} \delta'}{\sqrt{2} H} + F \cdot \delta' - qL \frac{\delta'}{2} = 0$

$$\left(V_{AB}\cdot\varphi_B - U_{AB}\cdot\delta_{AB} + W_{BA}\cdot\varphi_B - U_{BA}\,\delta_{AB}\right)\frac{\delta}{H} +$$

$$-\left(W_{BC}\varphi_B + V_{CB}\varphi_C - U_{BC}\delta_{BC} + \mu_{BC} + W_{CB}\varphi_C + V_{BC}\varphi_B - U_{BC}\delta_{BC} + \mu_{CB}\right)\frac{\delta}{L}$$

$$+\left(W_{CD}\varphi_C - U_{CD}\delta_{CD} + V_{CD}\varphi_C - U_{DC}\delta_{CD}\right)\frac{\sqrt{2}\,\delta}{\sqrt{2}H} + F\,\delta - qL\frac{\delta}{2} = 0$$

$$\left[\frac{V_{AB}+W_{BA}}{H} - \frac{W_{BC}+V_{BC}}{L}\right]\varphi_B + \left[-\frac{W_{CB}+V_{CB}}{L} + \frac{W_{CD}+V_{CD}}{H}\right]\varphi_C +$$

$$\left[-\frac{U_{AB}+U_{BA}}{H} - \frac{U_{BC}+U_{CB}}{L} - \frac{U_{CD}+U_{DC}}{H}\sqrt{2}\right]\delta + F - \frac{qL}{2} - \frac{\mu_{BC}+\mu_{CB}}{L} = 0$$

$$\left(-U_{BA}+U_{BC}\right)\varphi_B + \left(U_{CB} - U_{CD}\sqrt{2}\right)\varphi_C +$$

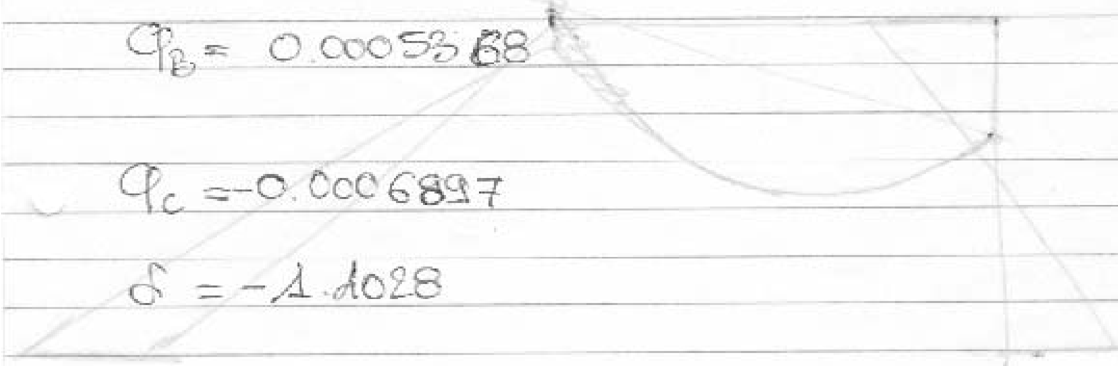
$$\left(\frac{U_{AB}+U_{BA}}{H} + \frac{U_{BC}+U_{CB}}{L} + \frac{U_{CD}+U_{DC}}{H}\sqrt{2}\right)\delta = F - \frac{qL}{2} - \frac{\mu_{BC}+\mu_{CB}}{L}$$

$$\begin{bmatrix} W_{BA}+W_{BC} & V_{BC} & U_{BC}-U_{BA} \\ V_{BC} & W_{CB}+W_{CD} & U_{CB}-\sqrt{2}\,U_{CD} \\ U_{BC}-U_{BA} & U_{CB}-\sqrt{2}\,U_{CD} & \dfrac{U_{AB}+U_{BA}}{H}+\dfrac{U_{BC}+U_{CB}}{L}+\dfrac{U_{CD}+U_{DC}}{H}\sqrt{2} \end{bmatrix} \begin{bmatrix} \varphi_B \\ \varphi_C \\ \delta \end{bmatrix} = \begin{bmatrix} -\mathcal{M}_{BC} & -\mathcal{M}_{CB} & +F-\dfrac{qL}{2}-\dfrac{\mathcal{M}_{BC}+\mathcal{M}_{CB}}{L} \end{bmatrix}^T$$

$$\begin{bmatrix} 2.1475\cdot 10^{11} & 6.23\cdot 10^{10} & -2.271\cdot 10^{7} \\ 6.23\cdot 10^{10} & 1.3188\cdot 10^{11} & 1.001\cdot 10^{6} \\ -2.271\cdot 10^{7} & 1.001\cdot 10^{6} & 71322.3 \end{bmatrix} \begin{bmatrix} \varphi_B \\ \varphi_C \\ \delta \end{bmatrix} =$$

$$\begin{bmatrix} +1.04\cdot 10^{8} \\ -1.04\cdot 10^{8} \\ -115000 \end{bmatrix}$$

SOLUZIONE



$\varphi_B = 0.0005388$

$\varphi_C = -0.0006897$

$\delta = -1.1028$

MOMENTI

$$M_{AB} = V_{AB} \cdot \varphi_B - U_{BA} \cdot \delta_{AB} = 9.015 \cdot 10^{10} \cdot 0.0005368 +$$

$$- 6.01 \cdot 10^7 (-1.4028) = 1.327 \cdot 10^8 \text{ Nmm} = 132.7 \text{ KNm}$$

$$M_{BA} = W_{BA} \cdot \varphi_B - U_{BA} \cdot \delta_{BA} = 9.015 \; 10^{10} \cdot 0.0005368 +$$

$$- 6.01 \cdot 10^7 \cdot (-1.4028) = 1.327 \cdot 10^8 \text{ Nmm} = 132.7 \text{ KNm}$$

$$M_{BC} = W_{BC} \varphi_B + V_{BC} \cdot \varphi_C - U_{BC} \cdot \delta_{BC} + \mu_{BC} = 1.246 \cdot 10^{11} \cdot 0.0005368 +$$

$$+ 6.23 \cdot 10^{10} \cdot (-0.0006897) - 3.739 \cdot 10^7 (-1.4028) (-1) +$$

$$-1.0417 \cdot 10^8 = -132.7 \text{ kNm}$$

$$M_{CB} = W_{CB} \cdot \varphi_C + V_{CB} \cdot \varphi_B - U_{CB} \cdot \delta_{BC} + \mu_{CB} = 1.246 \cdot 10^{11} \cdot (-0.0006897)$$

$$+ 6.23 \cdot 10^{10} (0.0005368) - 3.739 \cdot 10^7 (-1.4028)(-1) + 1.0417 \cdot 10^8 =$$

$$- 774672 \text{ Nmm} = -0.775 \text{ kNm}$$

$$M_{CD} = W_{CD} \cdot \varphi_C - U_{CD} \cdot \delta_{CD} = 6.678 \cdot 10^{10} \cdot (-0.0006897) +$$

$$- 2.361 \cdot 10^7 \cdot \sqrt{2} \cdot (-1.4028) = 780739 = 0.780 \text{ KNm}$$

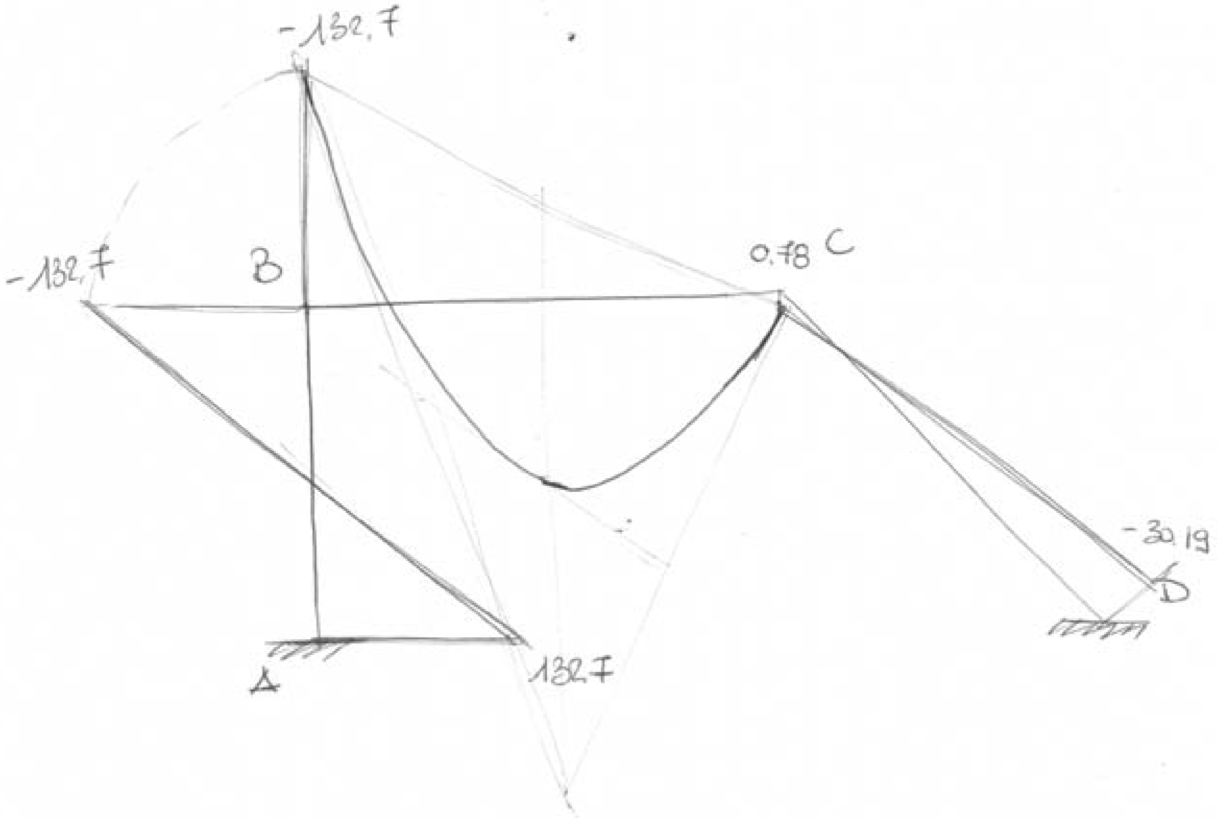
$$M_{DC} = V_{DC} \cdot \varphi_C - U_{DC} \cdot \delta_{DC} = 3.339 \cdot 10^{10} \cdot (-0.0006897) +$$

$$- 2.683 \cdot 10^7 \cdot \sqrt{2} (-1.4028) = 30.19 \text{ KNm}$$

Risulta

$$M_{CB} \simeq M_{CD}$$

per motivi di troncamento delle parti decimali.


-132,7
-132,7
B
0,78 C
-30,19
D
A
132,7